Multiresolution Representation and Reconstruction of Triangulated Surfaces

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OUTLINE

1. Shape representaton with triangle meshes
2. Level of detail: tradeoff between mesh complexity and accuracy
3. The multi-triangulation (MT): a general multiresolution model
4. Extracting a shape at variable resolution from an MT
5. Building an MT from different data sources
6. Conclusions

Shape representaton with triangle meshes

- an object with a complex shape can be represented with a mesh of triangles
- each triangle approximates a portion of object surface within a given *accuracy*
- triangle meshes are easy to represent, manipulate, and visualize
- different data sources: CAD, sampling (e.g., data from range scanners, medical data), simulation, ...

Remark:

- more accurate representation $\Rightarrow$ more triangles
- more triangles $\Rightarrow$ higher processing times

Ideas:

- adapting the accuracy to the needs of an application can improve efficiency
- the accuracy of representation can be variable over different portions of the object

Level of detail (LOD)

Basic idea: always using the simplest possible mesh that satisfies the accuracy required by a given application.

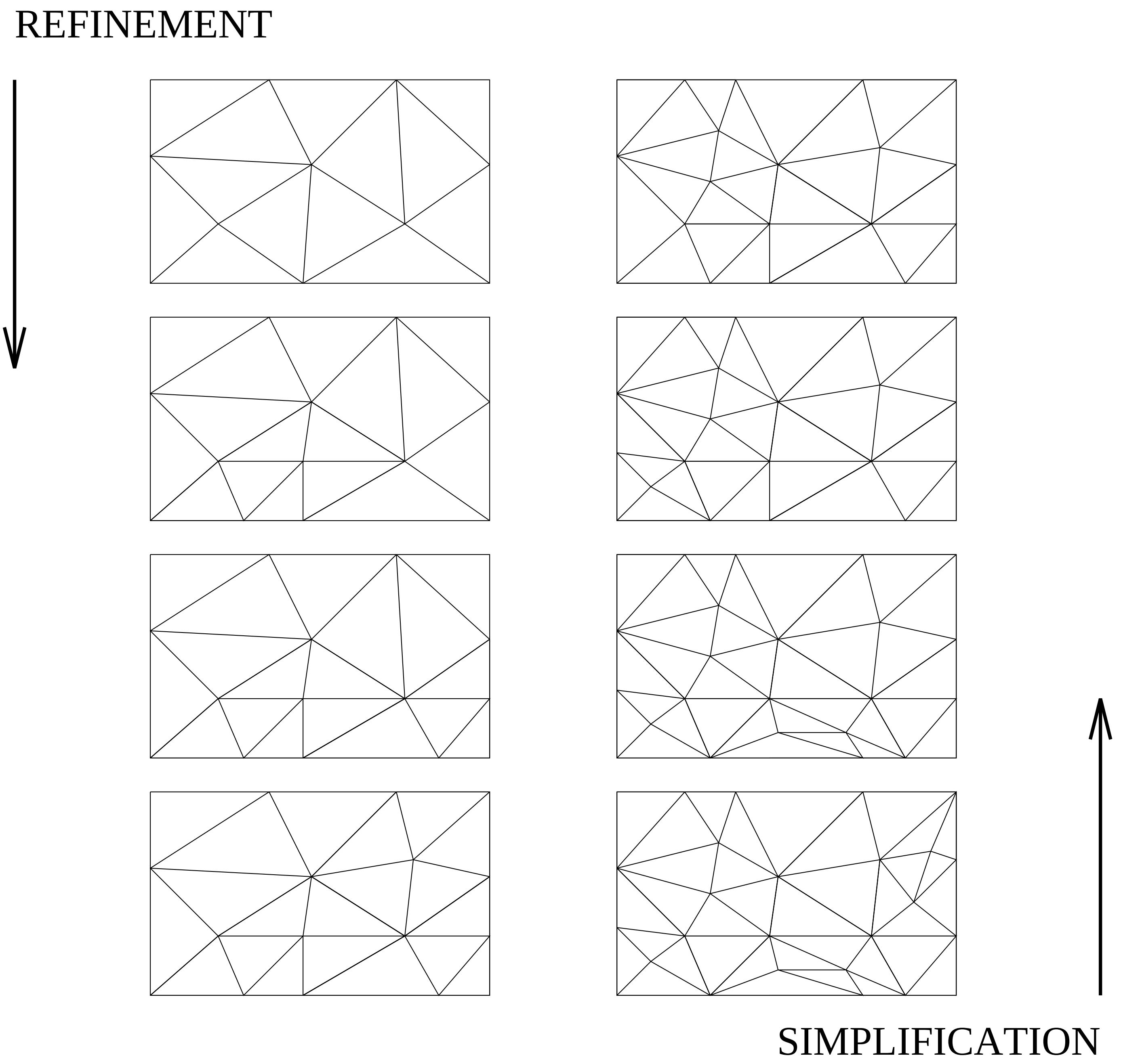
Alternative approaches:

- *on-the-fly simplification*: extract from raw data a mesh of minimal size whose accuracy is sufficient to the application needs.
  - there exist many simplification methods;
  - simplfication is usually an expensive task: no hope to perform it in real time except for some special cases.
- *multiresolution model*: build off-line a model that incorporates many different representations, and can be queried efficiently.
  - more expensive in terms of space
  - much more efficient: support to real time.

A naive LOD model

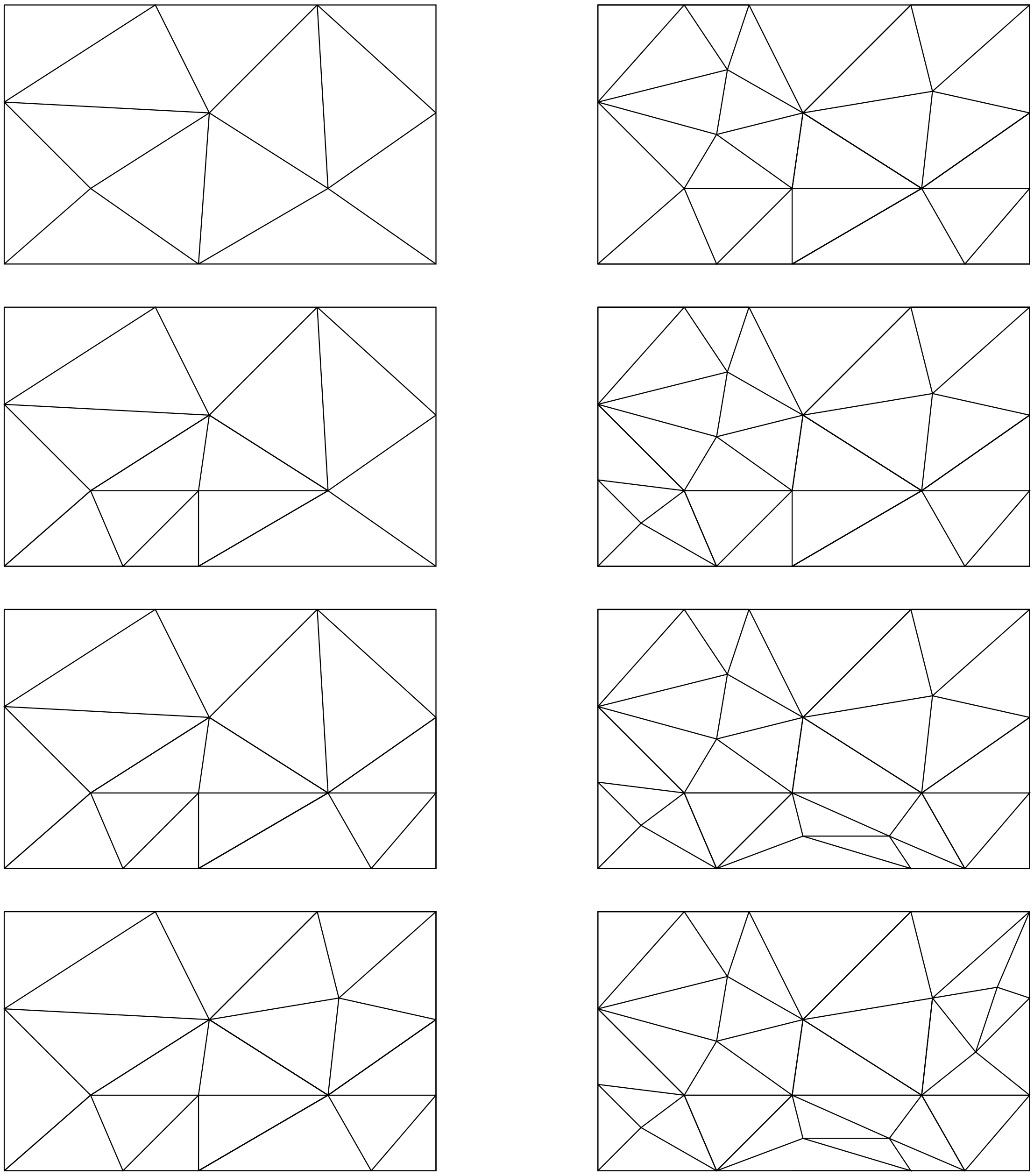
Basic idea: different accuracies of representation can be obtained through local modifications of a mesh.

A sequence of meshes is obtained by iterative application of local modification steps.



Remark: the model can be considered as formed by an initial mesh plus a sequence of local modifications (fragments)

Key observation: some fragments are *mutually independent*, i.e., one doesn't need the other to occur

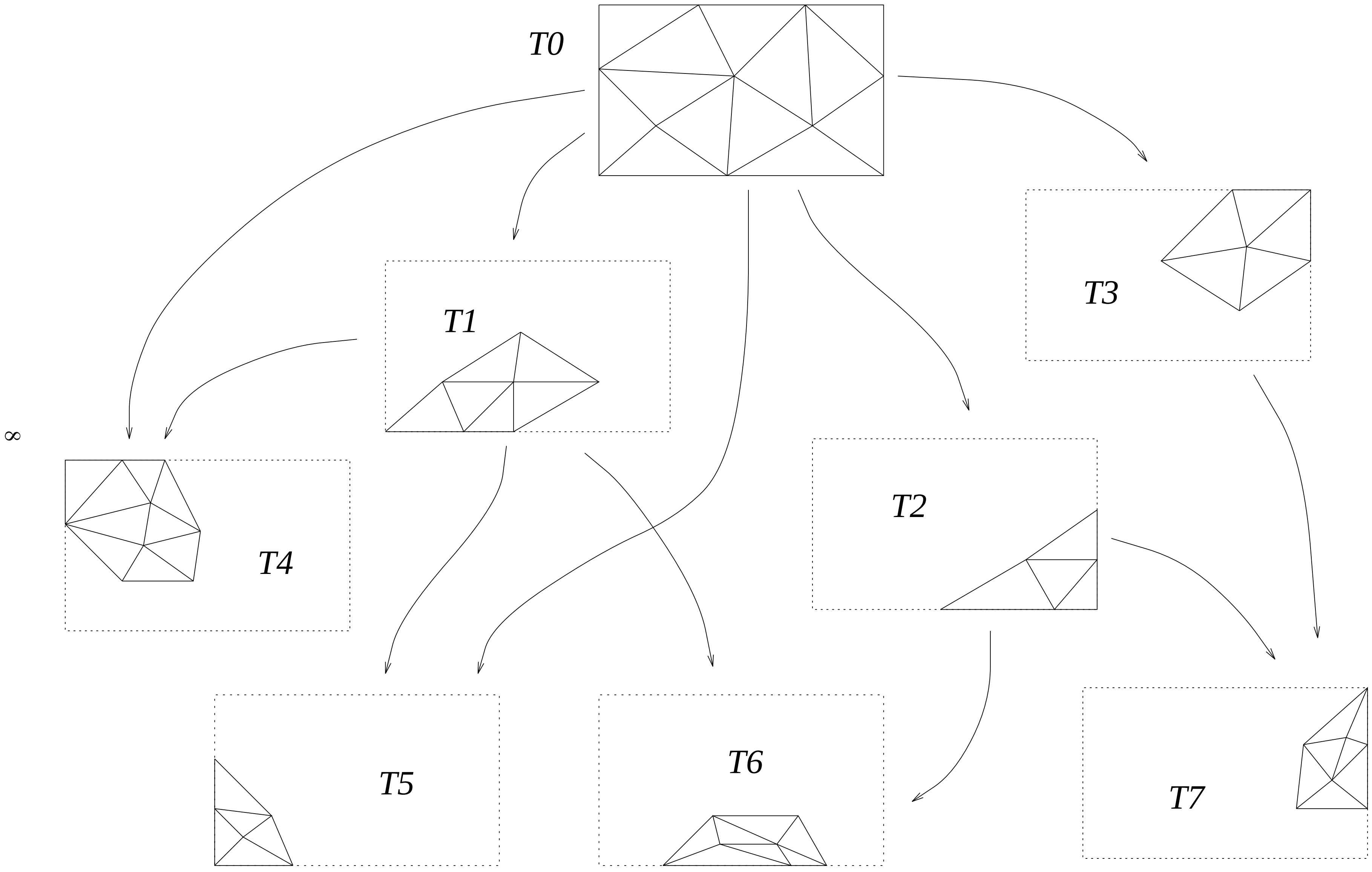


The Multi-Triangulation (MT)

Idea: we don't need a total order!

Fragments are arranged in a partial order:

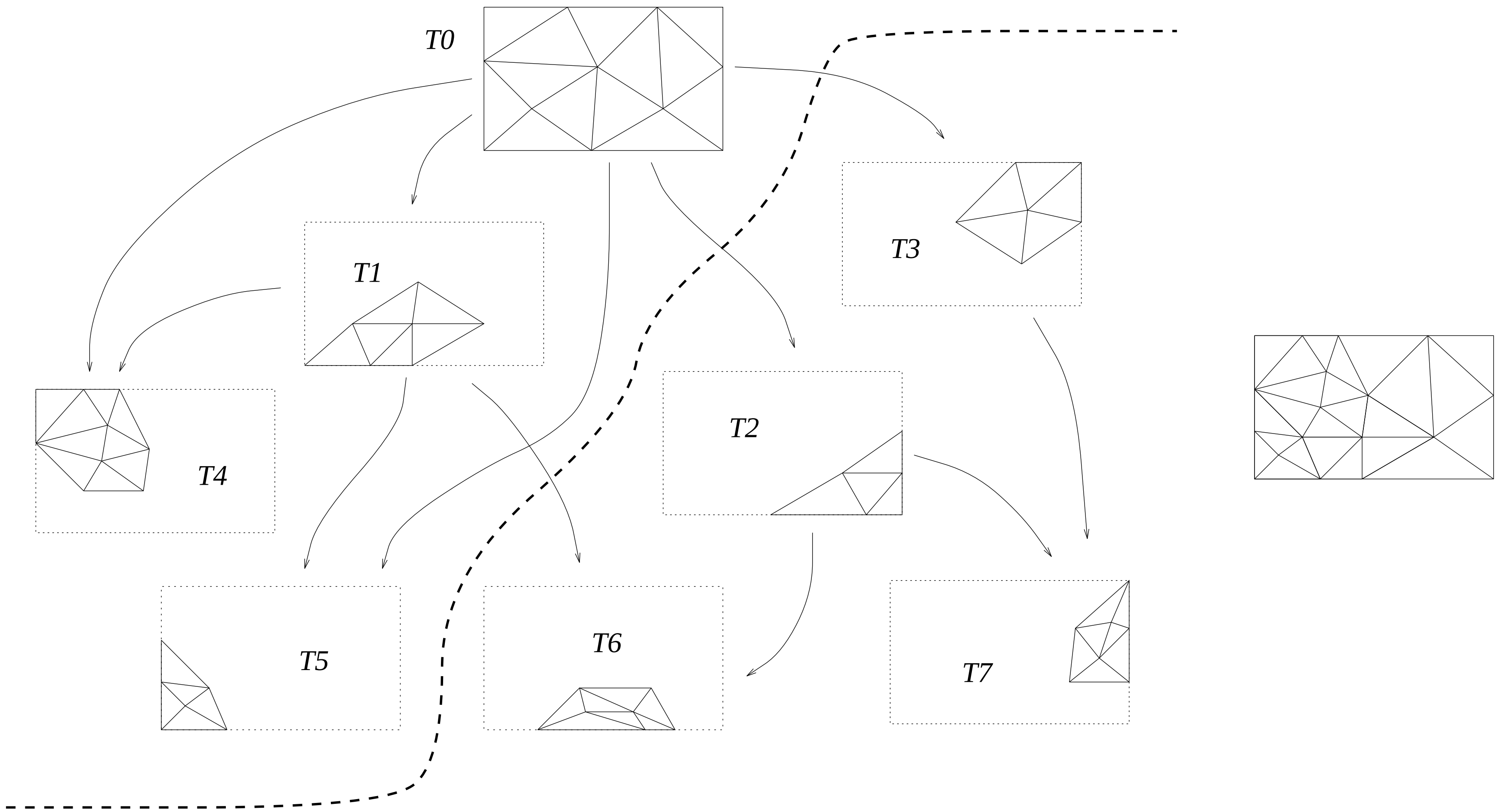
- the coarsest mesh is the minimal element
- fragment T_b follows fragment T_a if T_b refines some portion of T_a


T0
T1
T2
T3
T4
T5
T6
T7
∞

Extracting meshes from an MT

Besic ideas:

- given a *cut* in the MT, the combination of all fragments above the cut forms a mesh;
- if a cut is moved below a given fragment, the mesh is refined (i.e., becomes more accurate) over the area covered by that fragment.


T0
T1
T3
T2
T4
T5
T6
T7

Fundamental results [Puppo, 1996]:

Theorem 1 (expressive power): *any possible* mesh made of triangles of the MT corresponds to some cut.

Theorem 2 (efficiency): given a threshold function τ that bounds the maximum approximation error allowed at each point in space, *the smallest* mesh made of triangles of the MT, and satisfying τ can be found *in linear time*.

Possible extraction algorithms:

- **static**: given an MT, extract the minimal mesh satisfying a threshold function τ;
- **dynamic**: given a current cut on the MT, update the cut to obtain the minimal mesh satisfying a threshold function τ.

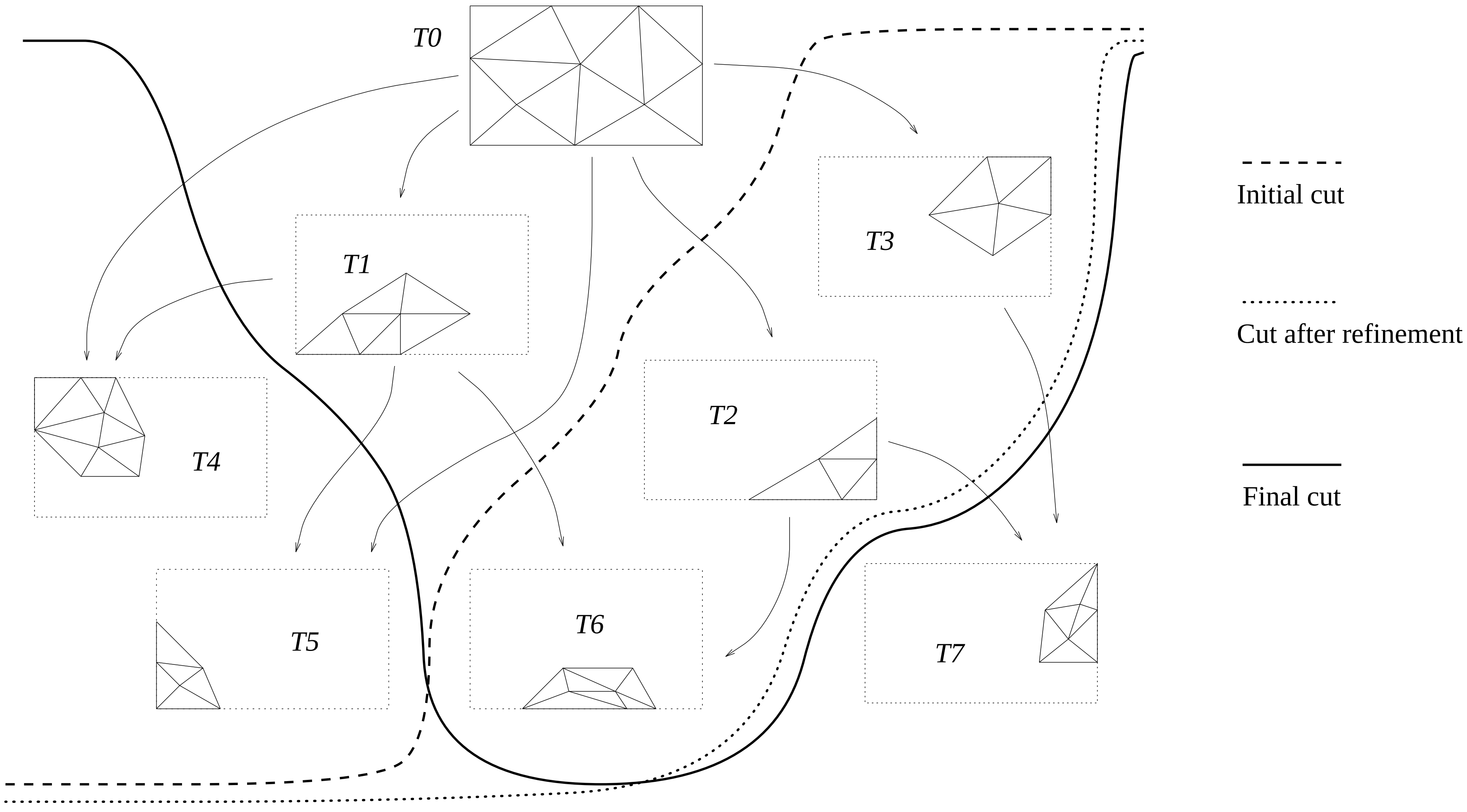
A dynamic extraction algorithm

Idea: given a current cut, and a new threshold:

- move the cut downward where the mesh is not refined enough;
- move the cut upward where the mesh is too refined.

Pseudo-code:

1. for every triangle t in the current mesh do
   if t doesn't satisfy τ then
   move the cut downward until the fragment refining t is above it;
2. while some leaf fragment in the current cut can be eliminated do
   move the cut above it.


T0
T1
T2
T3
T4
T5
T6
T7
Initial cut
Cut after refinement
Final cut

Building an MT

From sampled data:

- for height fields (terrains): MT decomposing the domain in the xy plane; triangles are lifted in 3D space by applying the sampled height at each vertex.
  - construction by refinement: start with a simple mesh covering the domain, and incrementally add data points; each point insertion makes a new fragment.
  - construction by simplification: start with a triangulation of all data points, and progressively simplify by decimation; each point deletion makes a new fragment.
- for manifold surfaces: build an initial mesh on all data points, then build the MT by simplification.

  Methods for building the initial mesh:
  - from scattered data: a reconstruction method is needed [Boissonnat 1984, Veltkamp 1993, Bajaj et al. 1995-1996]
  - from contours: the mesh is obtained by connecting contours lying on consecutive planes [Keppel 1985, Fuch et al. 1977, Boissonnat 1988, Geiger 1993]

- from CAD surfaces: data are a collection of adjacent parametric patches (trimmed surfaces)
  - build a 1D multiresolution model for each trimming curve (boundary of a patch), through refinement;
  - build an MT for each patch through refinement, in accordance with the 1D models of its boundaries;
  - weld models of different patches along their common boundaries;
  - the result is a unique MT of the whole surface.

Conclusions

- The MT is a powerful multiresolution structure that generalizes over previously known models
- The MT is suitable to any sort of shapes that can be modeled with triangle meshes
- A minimal mesh satisfying a given accuracy can be extracted in real time

- Prototype systems for real-time visualization of terrains and free-form surfaces (implemented)
- Prototype system for CAD surfaces (under implementation)
- Extension to volume data: Multi-Tetrahedization (under implementation)
- Possible extension to other techniques of shape representation that admit a multiresolution approach based on decomposition, and local updates (e.g.: skeletons)