Simplification of Morphological Representations of Shapes for Analysis and Visualization

Lidija Čomić

University of Novi Sad, Serbia

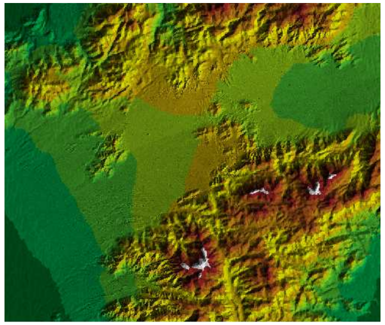
Maria Vitali

University of Genova, Italy

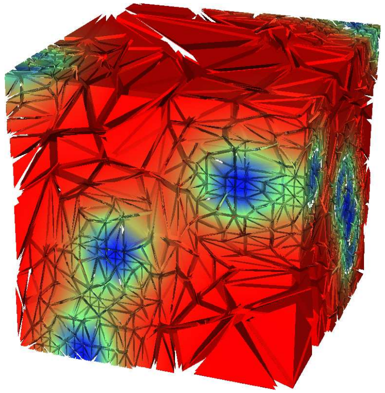
Introduction and Motivation

Morphological representation and simplification of scalar fields in nD.

Examples:



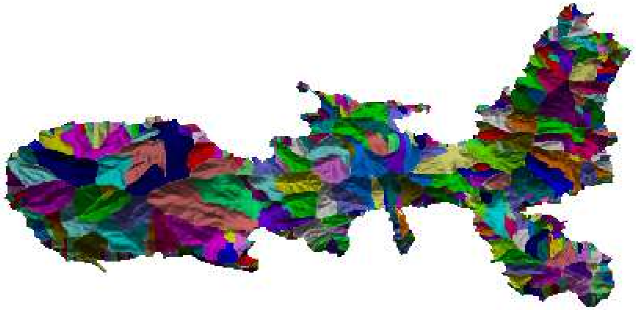
- terrains



- volume data (data from scientific simulation, medical data sets)
- time-varying volume data sets

Introduction and Motivation

* Morphological representation (segmentation)
* Simplification
* Hierarchical representation



Outline

- $*$ Morse Theory, Cancellation
- $*$ Removal and Contraction on Morse Complexes (Lidija)
- $*$ Simplification in discrete 2D scalar fields (Maria)

Outline

* **Morse Theory**, Cancellation
* Removal and Contraction on Morse Complexes
* Simplification in discrete 2D scalar fields

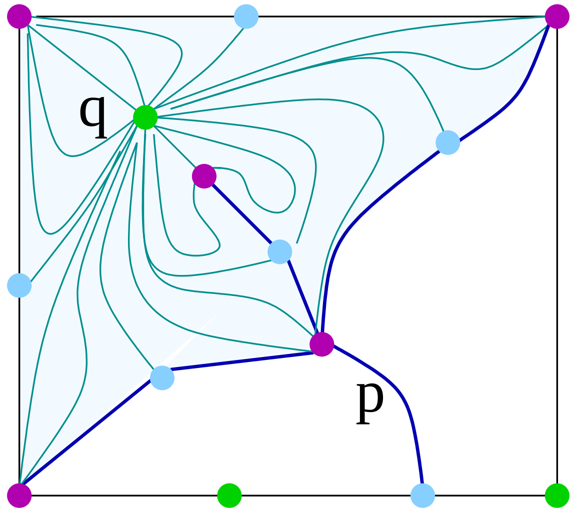
Morse theory

- $p \in \mathbb{M}$ is a *critical point* of f if $\nabla f(p) = 0$
- p is non-degenerate if the Hessian matrix at p is non-singular
- $f(x_1, x_2, \ldots, x_n) = f(p) - x_1^2 - \ldots - x_i^2 + x_{i+1}^2 + \ldots + x_n^2$
- i is the *index* of p, p is an i-saddle
- 0-saddles are called minima, n-saddles are called maxima
- Integral line is everywhere tangent to ∇f
- Each integral line connects two critical points

Descending Morse Complex

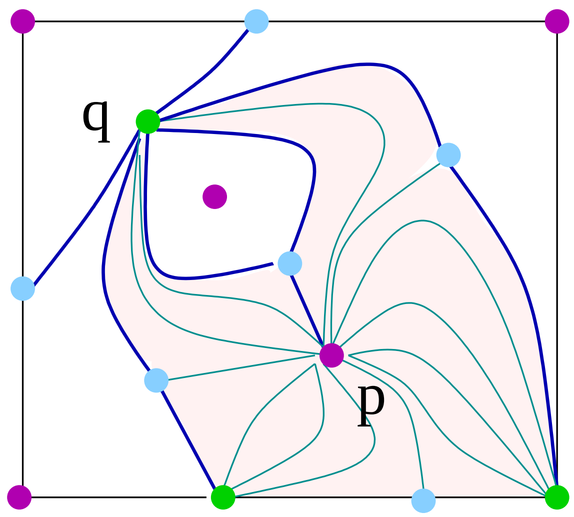
Integral lines converging to an i-saddle p form the *descending* i-cell of p.

Descending cells subdivide M into a *descending Morse complex*.



Ascending Morse Complex

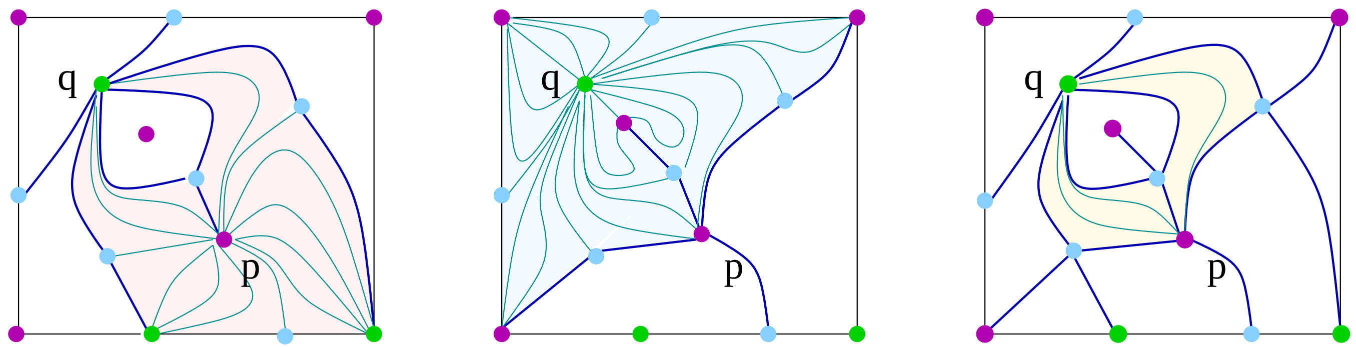
Integral lines originating at an i-saddle p form the *ascending* $(n-i)$-cell of p.
Ascending cells subdivide M into an *ascending Morse complex*.



Morse-Smale Complex

Function f is Morse-Smale if ascending and descending complexes intersect transversally.

In 2D: there is no saddle-saddle connection.



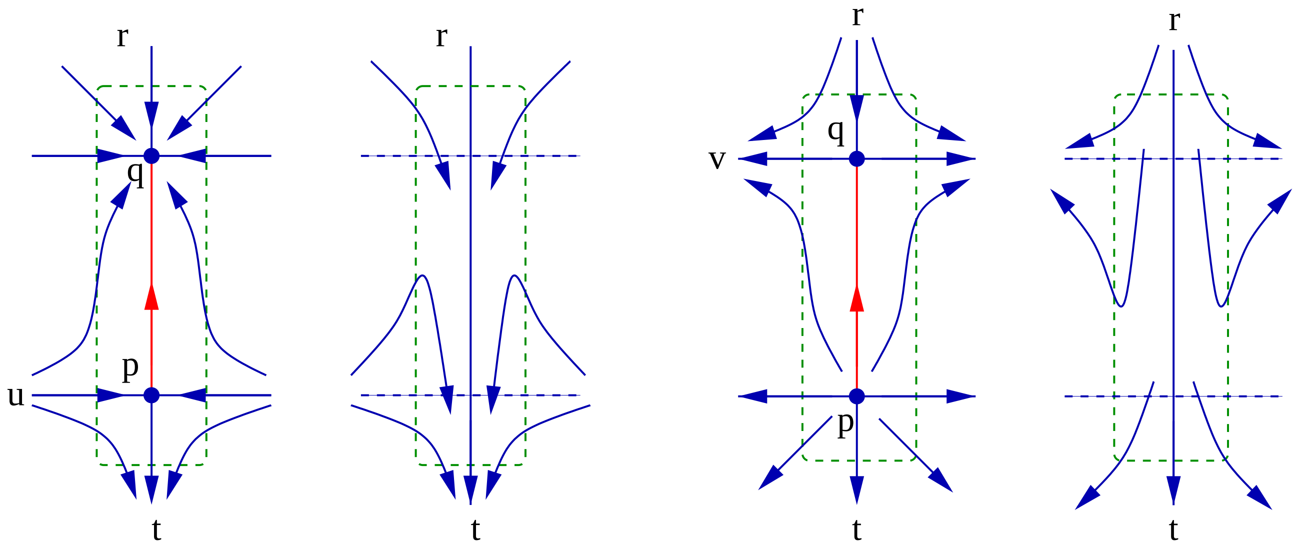
Morse-Smale complex is an overlay of the two Morse complexes.

Outline

* Morse Theory, **Cancellation**
* Removal and Contraction on Morse Complexes
* Simplification in discrete 2D scalar fields

Cancellation on Critical Points

i-saddle p and $(i+1)$-saddle q can be canceled if there is a unique integral line connecting them [Matsumoto 2002].



lines converging to q lines originating at p

Cancellation on Morse and Morse-Smale complexes

After cancellation of i-saddle p and $(i+1)$-saddle q

- $*$ each cell t in the co-boundary of p becomes incident to each cell r on the boundary of q in the descending Morse complex,
- $*$ each such pair (t,r) determines a new cell in the Morse-Smale complex.

t is unique if p is a minimum and q is a 1-saddle.

r is unique if q is a maximum and p is an $(n-1)$-saddle.

Cancellation in 2D

In 2D, there are three kinds of critical points:

* minima, saddles, maxima,

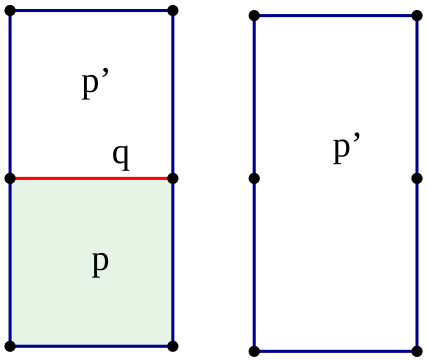
and two kinds of cancellations:

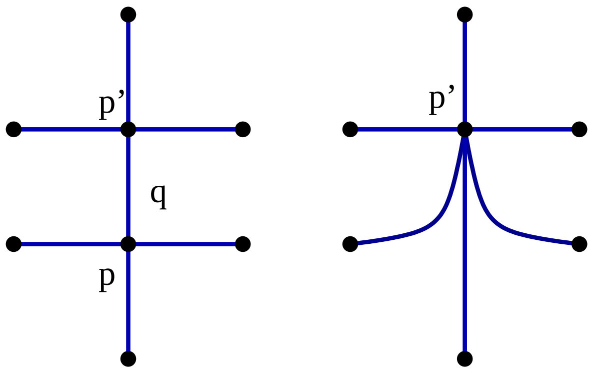
* minimum-saddle cancellation, maximum-saddle cancellation.

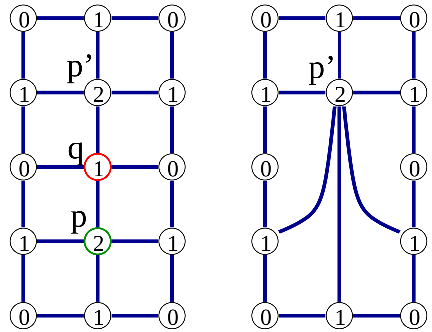
The two cancellations are dual to each other.

Cancellation in 2D

Maximum-saddle cancellation. q is a saddle, p and p' are maxima.



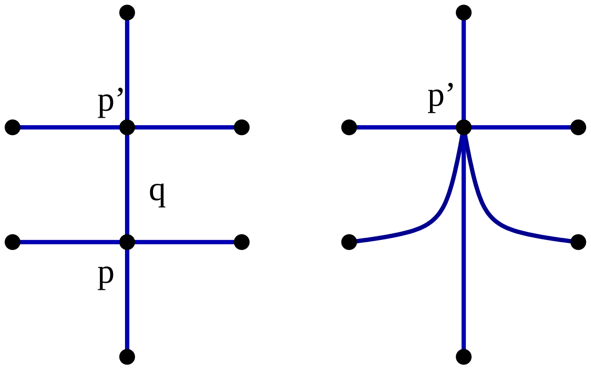




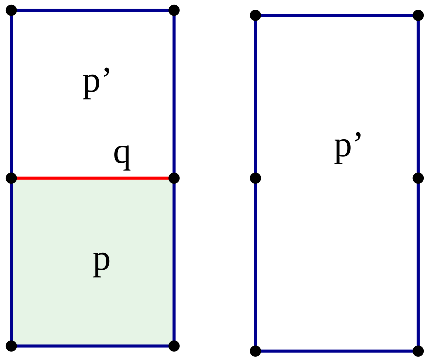
descending ascending Morse-Smale

Cancellation in 2D

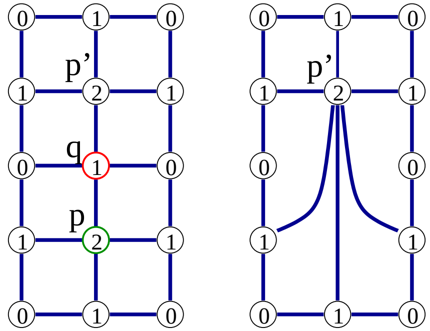
Minimum-saddle cancellation. q is a saddle, p and p' are minima.



descending



ascending



Morse-Smale

Cancellation in 3D

In 3D, there are four kinds of critical points:

* minima, 1-saddles, 2-saddles, maxima,

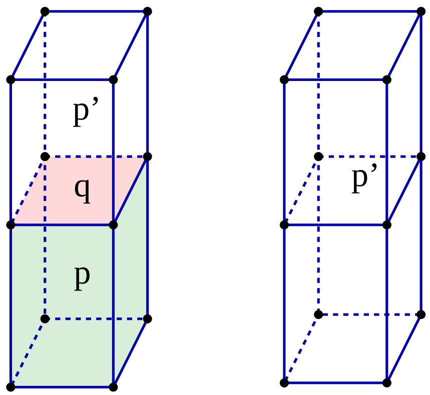
and three kinds of cancellations:

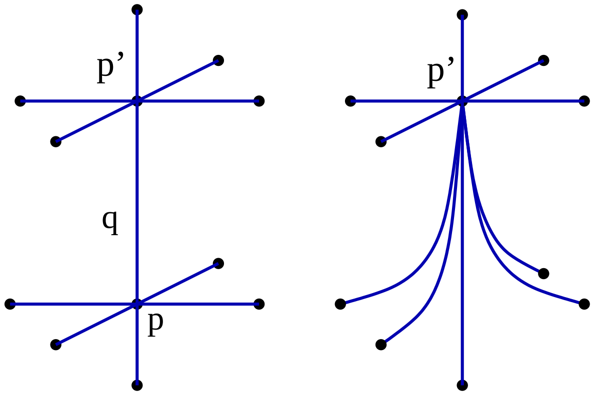
* minimum-1-saddle cancellation, maximum-2-saddle cancellation, 1-saddle-2-saddle cancellation.

Minimum-1-saddle cancellation is dual to maximum-2-saddle cancellation.

Cancellation in 3D

Maximum-2-saddle and minimum-1-saddle cancellation.





q is a 2-saddle

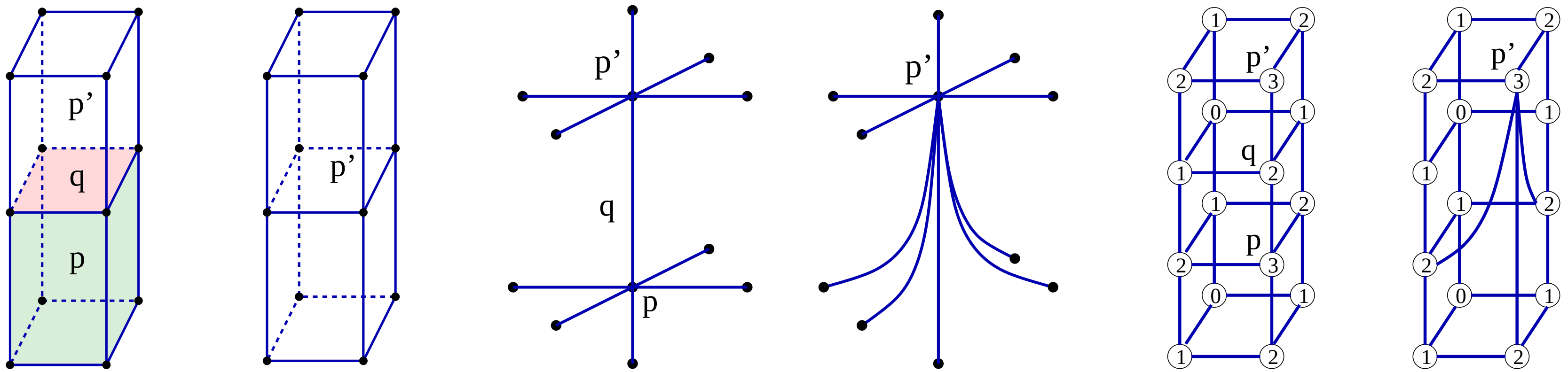
p and p' are maxima

q is a 1-saddle

p and p' are minima

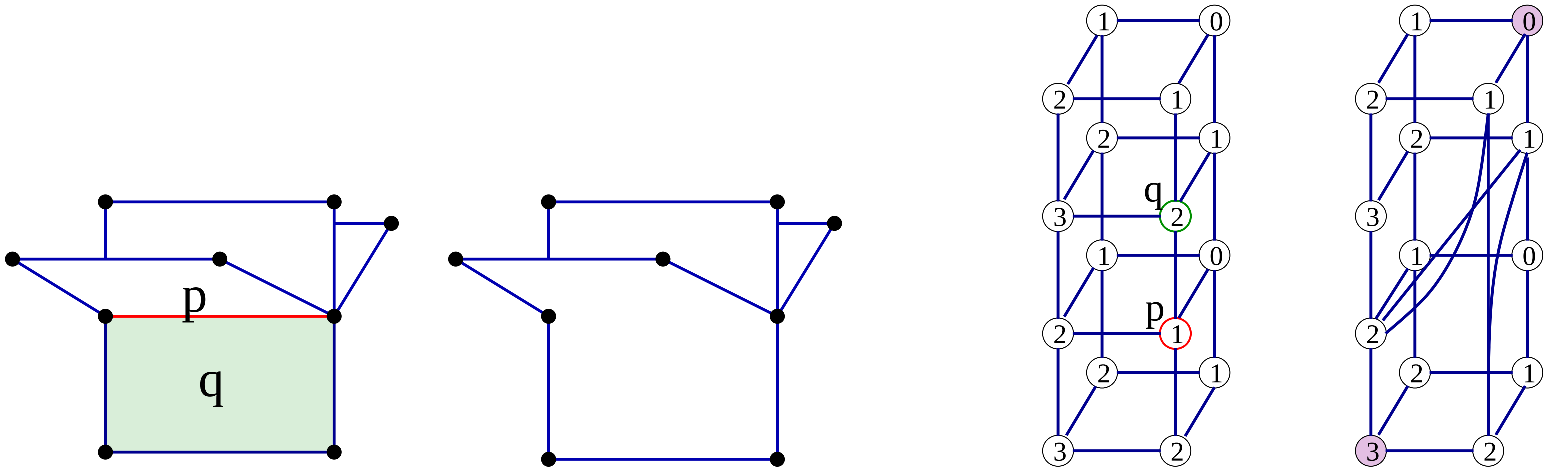
Cancellation in 3D

Effect of maximum-2-saddle and minimum-1-saddle cancellation on the Morse-Smale complex



Cancellation in 3D

Cancellation of 1-saddle p and 2-saddle q.



- not a simplification operator
- increases the number of cells in the Morse-Smale complex

Outline

* Morse Theory, Cancellation
* **Removal and Contraction on Morse Complexes**
* Simplification in discrete 2D scalar fields

Removal and Contraction on Morse Complexes

* elementary operators
* defined in arbitrary dimensions
* simplify Morse and Morse-Smale complexes
* minimally complete set of operators

Removal and Contraction on Morse Complexes

A *removal* (of index i) of an $(i+1)$-saddle p and an i-saddle q is defined if i-saddle q is connected to

$*$ exactly two different $(i+1)$-saddles p and p'

Dually, a *contraction* (of index $i+1$) of an i-saddle p and an $(i+1)$-saddle q, is defined if q is connected to

$*$ exactly two different i-saddles p and p'

Removal and Contraction in 3D

Removal of index 2 is equivalent to a cancellation of a maximum p and 2-saddle q.

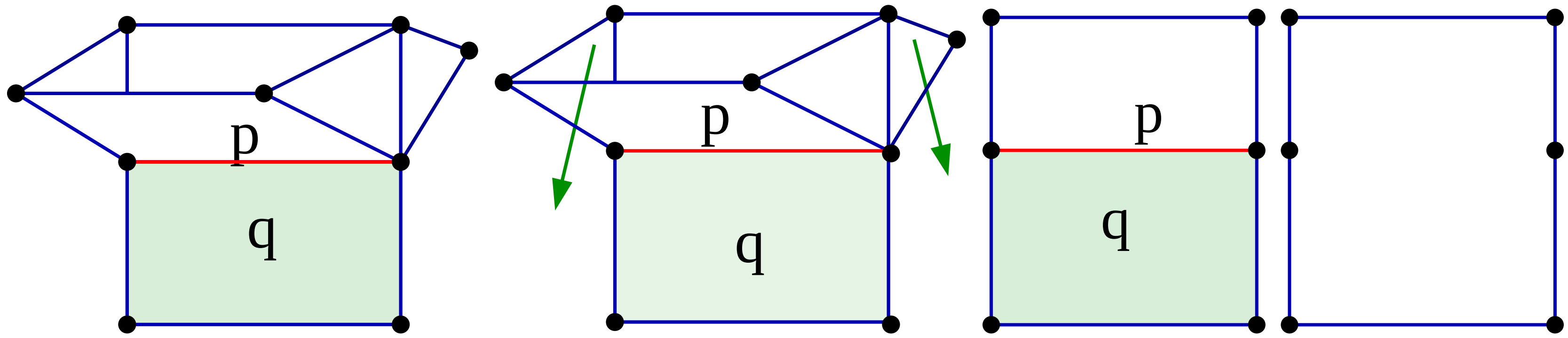
Contraction of index 1 is equivalent to a cancellation of a minimum p and 1-saddle q.

Removal of index 1 in 3D: remove 1-saddle q and merge 2-saddle p into 2-saddle p'

Contraction of index 2 in 3D: contract 2-saddle q and collapse 1-saddle p into 1-saddle p'

Cancellation through Removals

Cancellation of 1-saddle p and 2-saddle q.



Removal and Contraction as Euler Operators

Removal of index i, and contraction of index $i+1$:

* $*$ delete an i-cell and an $(i+1)$-cell from a descending Morse complex
* $*$ delete an $(n-i)$-cell and an $(n-i-1)$-cell from an ascending Morse complex

Removal and Contraction as Euler Operators

They maintain the Euler-Poincare formula

$$\sum_{i=0}^{n}(-1)^i\beta_i = \sum_{i=0}^{n}(-1)^i c_i.$$

- β_i is the i-th Betti number
- c_i is the number of i-cells

They form a basis of a set W of topological operators for modifying Morse complexes.

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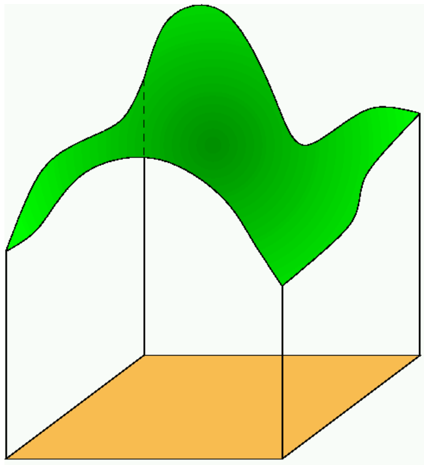
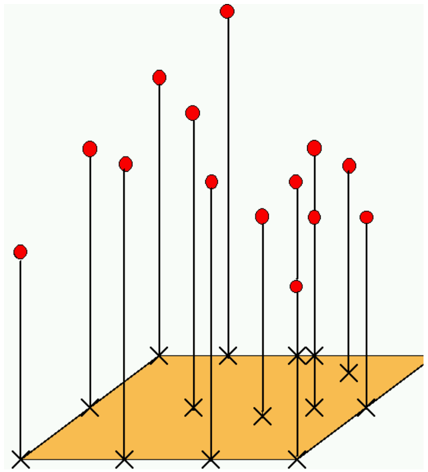
* Morse Theory, Cancellation
* Removal and Contraction on Morse Complexes
* **Simplification in discrete 2D scalar fields**

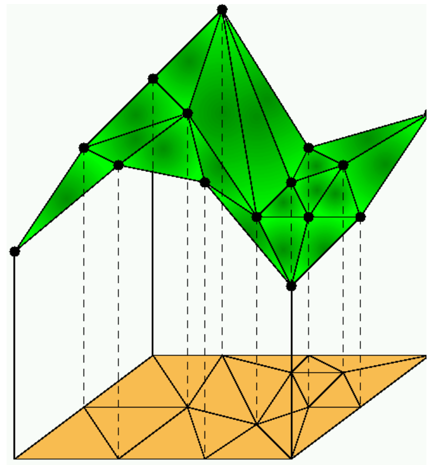
Discrete 2D scalar field

In the discrete, we represent a 2D scalar field through:

- set of points in R^2. Terrain function z = f(x,y).
- Triangulated Irregular Network (TIN). Piecewise linear interpolation.



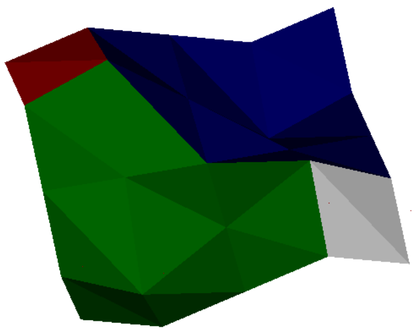


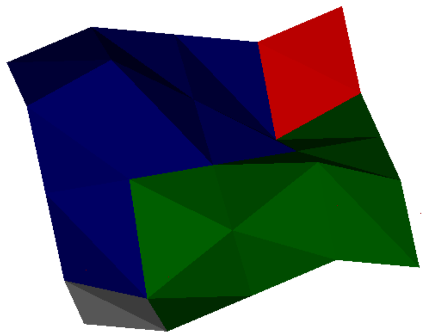


Morse Theory in the Discrete

Discrete Morse theory (Forman 1998) transposes Morse theory from continuum to a combinatorial context

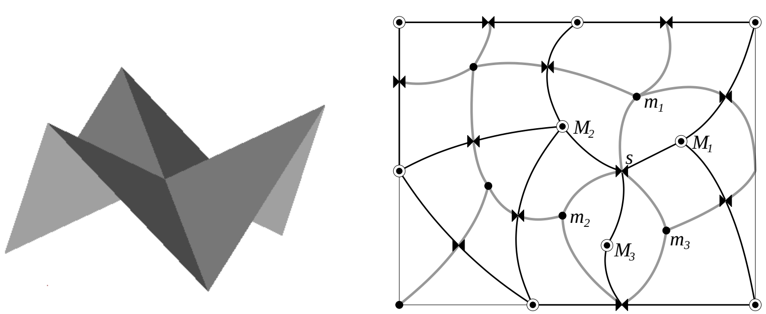
Piece-wise Linear Morse theory (Banchoff 1970, Edelsbrunner et al 2001) mimics the behavior of Morse functions in a discrete context as critical points and Morse complexes.



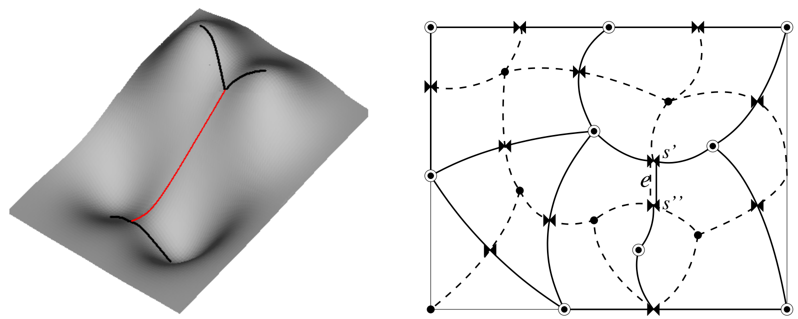


Morse Theory in the Discrete

Commonly available real data do not satisfy the Smale condition, (i.e.,ridges and valleys do not intersect only in a saddle).



Multiple saddles

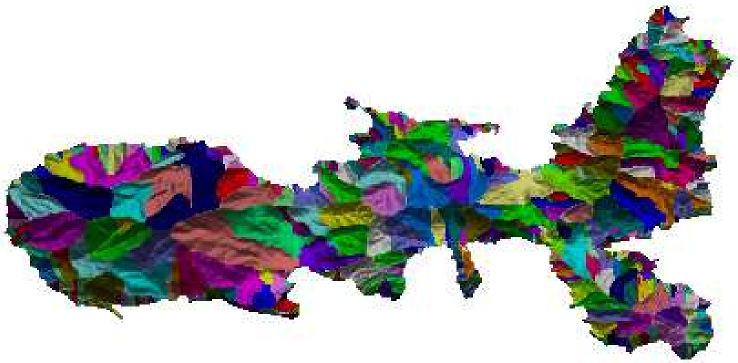


Macro-saddles

Morse Theory in the Discrete

Datasets are very large, usually affected by noise. Our purpose:

- simplification
- multiresolution representation

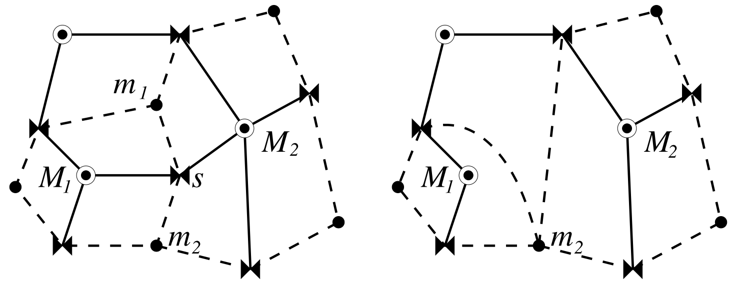


Simplification operators

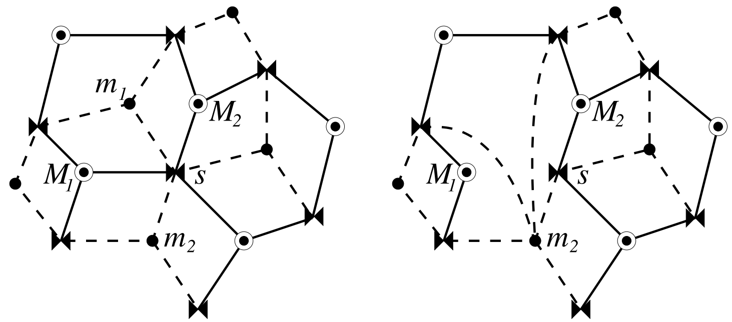
We extend simplification operators:

minimum - saddle

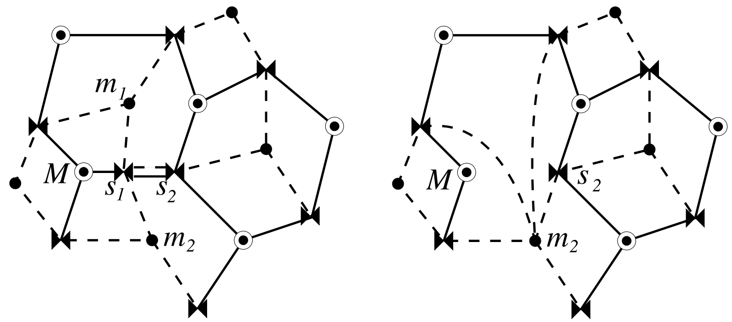
maximum - saddle



to handle multiple saddles:



to handle macro-saddles:



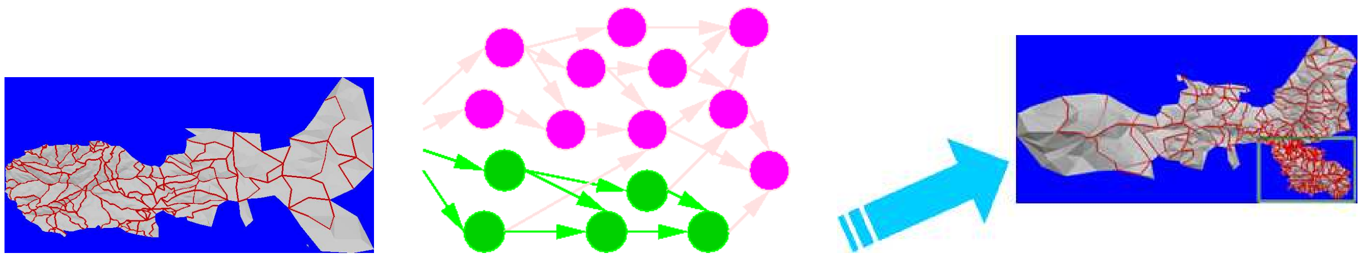
Multiresolution Morphological Model

Based from the Morse complexes at full resolution through simplification operators.

Elements of the model:

Coarse representation (final result of simplification)

Refinement steps that allow reconstructing the initial full-scale model (inverse of performed simplification steps)



Extraction of adaptative morphological representations.

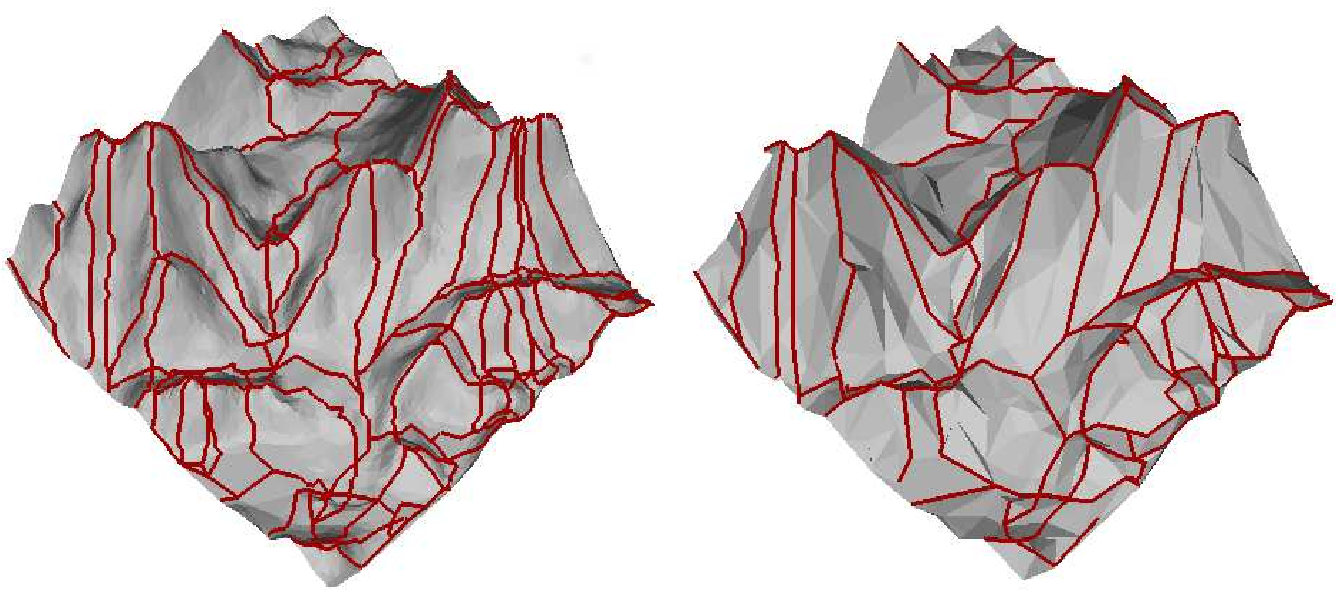
Multiresolution Morse Triangulation

Multiresolution Morse Triangulation consists of:

- multiresolution morphological model,
- multiresolution geometrical representation.

Results

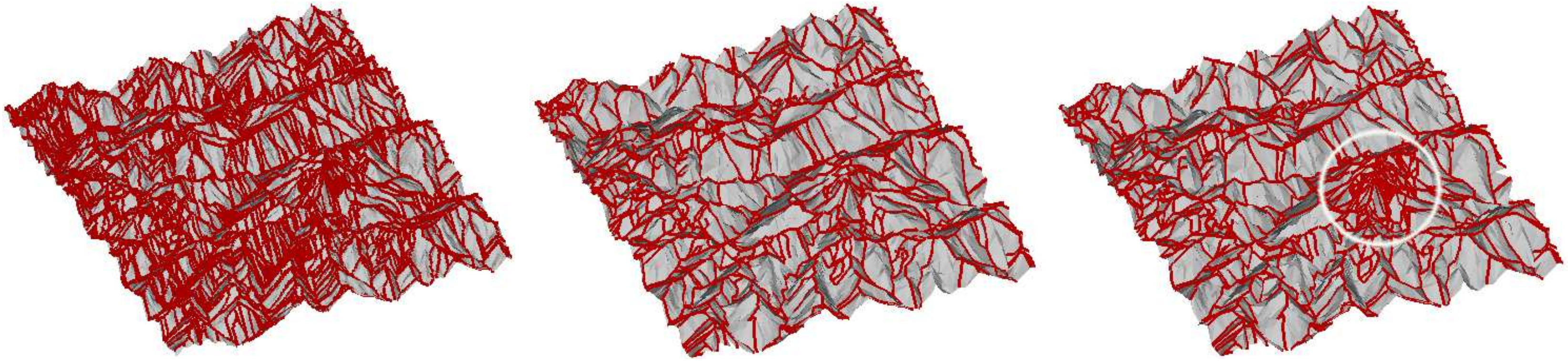
Mount Marcy mesh.



At full resolution (left). Extraction with a threshold of 10 meters on persistence (right).

Results

Monviso mesh.



Original decomposition of Monviso (left), representation extracted from the MMT at uniform resolution with a persistence threshold of 200 meters (middle), at variable resolution with a region of interest covering 1/16 of the domain (right).

Acknowledgment

We would like to thank Leila De Floriani and Paola Magillo for their help in preparing this presentation.

The End

Thank you for your attention.