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Optimal initial margin levels in South African futures markets: An empirical analysis

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1. Introduction

Futures margins are initial deposits ensuring investors perform according to the terms of the futures contract. Long contracts on futures are contracts to buy the underlying instrument at the maturity date and are taken out when the purchaser expects the price of the underlying instrument at maturity to be more than the current futures price. Theoretically, at maturity the purchaser will hope to buy the underlying instrument at the contracted futures price and make a profit. The futures seller is at risk if the underlying instrument at maturity is cheaper than the contracted futures price, and the buyer does not have the money to pay the difference to the seller. The seller therefore has to guard against drops in the price of the underlying instrument. The buyer has to pay an initial margin equal to the expected drop in price at a certain level of probability for that particular day. Thereafter, the margin is adjusted on a daily basis depending on the daily mark-to-market movement of the price.

Conversely, short contracts on futures are contracts to sell the underlying instrument at the maturity date and are taken out when the purchaser expects the price of the underlying instrument at maturity to be less than the current futures price. The futures seller has to guard against the price of the underlying instrument at maturity being more than the contracted futures price. Initial margins will be based on expected price rises in this case. In determining initial margins, the knowledge of probable positive and negative price changes in underlying instruments is therefore essential.

The South African Futures Exchange (SAFEX) sets its initial margins on simple unhedged contracts to cover 99,95% of all expected price variations. SAFEX takes the approach that the natural logarithms of price changes on the Johannesburg Stock Exchange (JSE) All Share (ALSI), All Gold (GLDI), and Industrial (INDI) Indices are normally distributed. Support for this approach can be found in Figlewski (1984:400) and Gay, Hunter and Kolb (1986:311). However, evidence exists that distributions of price changes (percentage logarithmic) are leptokurtic, as indicated by the research of Longin (1994), meaning that extreme price changes occur more frequently than indicated by the normal model. Extreme movements in futures prices are, however, the crux of the problem of optimal margin setting, as only large price variations cause brokers to suffer losses (Edwards and Neftci, 1988:629).
A different parametric method of describing the tails (extremes) of the distributions of price changes will be used to find optimal margins for the SAFEX futures contracts on the ALSI, GLDI, and INDI for the years 1986 to 1998. The results of this method, developed by Longin (1994), will be compared with empirical results, normal distribution results, and historical SAFEX margins.

Section 2 introduces this method of determining price changes, and consequently margins, based on extreme value theory. Section 3 compares the results of extreme value and normal methods of margin setting by:

- testing the suitability of the normal distribution for explaining log price changes on ALSI, GLDI, and INDI from 1986 to 1998, using Chi-square tests;
- testing the appropriateness of the Frechet extreme value distribution for explaining log price changes on ALSI, GLDI, and INDI from 1986 to 1998;
- comparing the margins calculated by the normal method, by the Frechet extreme value method, and empirically at the SAFEX required level of margin violation probability; and
- comparing the Frechet margins with the SAFEX volatility-adjusted normal distribution margins, and determining which are more appropriate in the light of empirically observed price changes.

Conclusions are drawn in Section 4.

2. Margin setting by Extreme Value Theory

A margin is sought for a time period of n trading days, so that the probability of daily violation is p over n trading days, at margin level r. This is opposed to the normal distribution method where margin violation over every single day is considered. A probability of violation over a number of trading days is considered so that extreme value theory, which gives the asymptotic distribution of minima and maxima random price changes, can apply. Taking the minima and maxima price changes over a number of days ensures that only volatility due to large and extreme price variations is taken into account, which is what margin setting aims to cater for (Jansen and De Vries, 1991:18).

The most important information about the extremes is contained in the tails of the distribution. If MINn is the lowest daily price change, and MAXn the highest daily price change observed over n trading days, the asymptotic behaviour of the MINn and MAXn are studied in extreme value theory.

The type of extreme value distribution obtained is determined by the tail index, τ(Longin, 1994:10). The larger negative τ is, the more extreme values there are. When $\tau = 0$, a double exponential Gumbel distribution applies, $\tau < 0$gives rise to a Frechet distribution, and $\tau > 0$ leads to a Weibull distribution. The other parameters in the extreme value distribution are α and β, respectively the scale (dispersion), the location (average) parameters.

In order to estimate the unknown parameters, the procedure as described by Gumbel and Kinnison (Longin, 1994:11) ensues.

Let the random percentage logarithmic daily price change be R. Firstly the extremes are selected from the data: After n units of time there are n observations viz. $R_1, R_2, \ldots, R_n$, from which the smallest observation taken is $MINn_1$. From the next n observations a minimum called $MINn_2$is obtained. If there are a total of n.N observations, then there are N minima, $MINn_1$, $MINn_2$, … $MINn_N$.

Secondly the above sequence is arranged in increasing order to obtain the order statistic $MIN'n_1 < MIN'n_2 < \ldots < MIN'n_N$.

Each of the above order statistics may be written in the form $MIN'n_m$where m <= N and represents the position of the particular minimum in the order. Frequencies are chosen to match with every order statistic. According to Kinnison (1985:90), each order statistic, $MIN'n_m$is matched with its frequency m/(N+1).

The procedure is repeated with the maximal price changes. The frequencies m/(N+1), in the case of both minimal and maximal order statistics, are matched smallest to smallest and largest to largest with the absolute values of the order statistics.

The extreme value equation regression parameters of τ, α and $\beta$are determined using the regression equations below:

for minimal price changes:

$$-\ln[-\ln(m/(N+1))] = (1/\tau^{min}).\ln\alpha_n^{min} - (1/\tau^{min}).\ln[\alpha_n^{min} - \tau^{min}.(\beta_n^{min} - MIN'_{n.m})] + u_m \quad \ldots(1)$$

for maximal price changes:

$$\ln[-\ln(m/(N+1))] = (1/\tau^{max}).\ln\alpha_n^{max} - (1/\tau^{max}).\ln[\alpha_n^{max} - \tau^{max}.(MAX'_{n.m} - \alpha_n^{max})] + u_m \quad \ldots(2)$$

The parameters of the regression equations are solved by the least squares method using the software package E-Views.

3. Empirical Results

3.1 Introduction

The share index prices of ALSI, GLDI, and INDI for February 1986 to February 1998 were used instead of the futures prices. Figlewski (1984:401) also uses this approach since the volatilities of share and futures prices match each other closely, especially for the near contract futures which converge to share prices as contract expiry approaches. (All comparisons will be made on near contract margins.) Furthermore, Warshawsky (1989:429) points out that arbitrage keeps share prices and futures prices closely in line. He, however, points out that when the market is moving rapidly, the futures market can lead the cash market. Futures price volatility is, therefore, occasionally greater than share index price volatility in periods of market crisis. Such instances will give rise to under-margining (Figlewski, 1984:413).

A time series was created of percentage logarithmic price changes, *R*, defined by Figlewski and Kofman (Longin, 1994:15) as

$$R_t = 100.\ln[(P_t / P_{t-1})] \qquad ...(3)$$

where P_t is the closing index price on day t , and P_{t-1} that of the day before.

The approach used expresses margins as percentage logarithmic margins. The percentage changes can be converted into Rand margins in the following way: If the futures contract price is R100, a Rand margin of R10 corresponds to a percentage rate of 9,53% [=100.ln(110/100)] for a short position, and a percentage rate of -10,53% [=100.ln(90/100)] for a long position.

3.2 Determining the suitability of the normal distribution to describe price changes

For the indices ALSI, GLDI, and INDI, over the period 1986 to 1998, the average daily logarithmic price changes, standard deviations of those changes and clear lepto-kurtosis of changes (in the case of the ALSI and INDI indices) are shown in Table 1.

This observation is borne out by the Chi-square tests on the normality of the price changes in ALSI, INDI, and GLDI, found in Tables 2, 3, and 4. The observed frequencies at average and extreme levels of price changes are much higher than those expected from a normal distribution. In each index's Chi-square test, the calculated Chi-square statistic, $\chi^2_{calculated}$, was considerably higher than the critical value, $\chi^2_{critical}$, representing the rejection threshold at a significance level of 0,001. The normal distribution is therefore unsuitable to predict those extreme price variations which margins are supposed to guard against.

Table 1: Percentage logarithmic daily price changes on the ALSI, GLDI, and INDI indices for the period February 1986 to February 1998

	ALSI	GLDI	INDI
Sample size	2993	2999	2999
Average	0,052007	-0,009173	,06453
Standard deviation	,087074	,195345	,903213
Minimum	12,52107	12,939148	13,346832
Maximum	,695553	2,48372	,930045
Range	9,216623	5,422868	0,276877
Kurtosis	9,775728	,665987	6,343587
Standardized Kurtosis	20,841261	9,801674	18,050692

Table 2: Chi-square test conducted to determine the suitability of a normal distribution description for the percentage logarithmic price changes in ALSI for the period February 1986 to February 1998

Lower limit	Upper limit	Observed frequency	Expected frequency	Chi-square
at or below	-3,0286	26	6,9	53,096
-3,0286	-2,3429	22	34,4	4,474
-2,3429	-1,6571	71	132,1	28,290
-1,6571	-0,9714	194	345,1	66,128
-0,9714	-0,2857	630	612,9	0,475
-0,2857	0,4000	1039	740,9	119,953
0,4000	1,0857	681	609,4	8,409
1,0857	1,7714	214	341,1	47,359
1,7714	2,4571	75	129,9	23,185
2,4571	3,1429	27	33,6	1,304
above 3,1429		14	6,7	8,014

$\chi^2_{calculated}$ = 360,686 $\chi^2_{critical}$ = 29,589 with 10 d.f. at significance level 0.001

Table 3: Chi-square test conducted to determine the suitability of a normal distribution description for the percentage logarithmic price changes in INDI for the period February 1986 to February 1998

Lower limit	Upper limit	Observed frequency	Expected frequency	Chi-square
at or below	-2,42857	29	8,7	47,77
-2,42857	-1,80000	23	49,8	14,42
-1,80000	-1,17143	72	198,2	80,38
-1,17143	-0,54286	302	495,0	75,24
-0,54286	0,08571	1090	775,9	127,16
0,08571	0,71429	1049	763,8	106,48
0,71429	1,34286	314	472,2	53,02
1,34286	1,97143	84	183,3	53,78
1,97143	2,60000	22	44,6	11,46

above 2,60000		14	7,5	5,65

$\chi^2_{calculated}$ = 575,349 $\chi^2_{critical}$ = 27,875 with 9 d.f. at a significance level of 0,0

Table 4: Chi-square test conducted to determine the suitability of a normal distribution description for the percentage logarithmic price changes in GLDI for the period February 1986 to February 1998

Lower limit	Upper limit	Observed frequency	Expected frequency	Chi-square
at or below	-6,4286	14	5,2	15,01637
-6,4286	-5,5714	11	11,7	0,04733
-5,5714	-4,7143	26	31,2	0,86667
-4,7143	-3,8571	54	71,3	4,19371
-3,8571	-3,0000	98	140,1	12,66424
-3,0000	-2,1429	185	236,9	11,38148
-2,1429	-1,2857	370	344,6	1,86890
-1,2857	-0,4286	529	431,2	22,17347
-0,4286	0,4286	573	464,2	25,50605
0,4286	1,2857	427	429,8	0,01863
1,2857	2,1429	308	342,4	3,45766
2,1429	3,0000	179	234,6	13,19803
3,0000	3,8571	97	138,3	12,34935
3,8571	4,7143	58	70,2	2,10488
4,7143	5,5714	31	30,6	0,00516
5,5714	6,4286	14	11,5	0,55138
above 6,4286		25	5,0	78,98063

$\chi^2_{calculated}$ = 204,384 $\chi^2_{critical}$ = 39,253 with 16 d.f. at a significance level of 0,001

3.3 The Estimation of the Asymptotic distributions of minimal and maximal price changes (Extreme Value Distributions)

The next step is to test the appropriateness of the Frechet extreme value distribution in describing price movements. The parameter estimates of the extreme value distribution were calculated for minimal and maximal price changes over intervals, n, of 30 days and 60 days on the ALSI, GLDI, and INDI for the period February 1986 to February 1998. All the regressions show an adjusted R^2 (coefficient of determination) of more than 0,975. The estimated parameter values are well estimated, and have all low standard errors, as indicated in the summary of results in Table 5. The tail index, τ, is consistently negative and significantly less than zero. In all instances of minimal and maximal price change distributions, the distributions are therefore of the Frechet-type (Longin, 1994: 16).

Table 5: A summary of the results of the Frechet distribution parameter estimates found in regressing ALSI, GLDI, and INDI logarithmic price changes for the period February 1986 to February 1998

Index

Interval	Min/Max	τ-Value (std. error)	α-Value (std. Error)	β-Value (std. error)	Log - likelihood	Sum of squared residuals	
	30 days	min	-0,475(0,017)	0,744(0,016)	-1,375(0,015)	34,165	2,956
ALSI	30 days	max	-0,223(0,015)	0,650(0,013)	1,533(0,012)	40,191	2,621
ALSI	60 days	min	-0,582(0,031)	0,940(0,033)	-1,881(0,029)	16,646	1,504
ALSI	60 days	max	-0,08(0,025)	0,721(0,022)	1,831(0,020)	21,335	1,247
GLDI	30 days	min	-0,192(0,008)	1,260(0,013)	-3,353(0,011)	119,166	0,540
GLDI	30 days	max	-0,078(0,010)	1,562(0,022)	3,742(0,019)	80,900	1,161
GLDI	60 days	min	-0,185(0,018)	1,529(0,034)	-4,052(0,028)	40,669	0,575
GLDI	60 days	max	-0,124(0,024)	1,665(0,049)	4,462(0,042)	24,720	1,089
INDI	30 days	min	-0,577(0,017)	0,632(0,013)	-0,937(0,011)	48,565	2,217
INDI	30 days	max	-0,361(0,015)	0,492(0,009)	1,218(0,009)	47,219	2,277
INDI	60 days	min	-0,667(0,032)	0,866(0,031)	-1,341(0,026)	17,903	1,430
INDI	60	max	-	0,553(0,01	1,477(0,01	19,253	1,355

	days		0,480(0,030)	9)	6)		

The regression results over the 30 day intervals are more favourable than over the 60 day intervals as evidenced by higher log-likelihood values and lower percentage standard errors. A 30 day interval would therefore be preferred in forming an extreme value distribution of price changes.

The results indicate that the magnitude and dispersion of minimum compared with maximum price changes in all three indices do not appear to be very different (α_i^{min} and β_i^{min} are not very different from α_i^{max} and β_i^{max}). However, there are more negative than positive extreme price changes in all three indices (τ^{min} is much lower than τ^{max}).

3.4 Calculating optimal margin levels

The initial margins calculated from the above data will be for unhedged accounts assuming a grace period of one day to pay the margin required. (In practice this can be much longer: the time period before cancellation of contract due to margin default is often effectively more than five days, and can vary per customer). Three methods of calculating margins are compared: margins based on the normal distribution of price changes; margins based on the Frechet distribution (or Weibull where $\tau > 0$); and margins based on average empirical price changes over time.

3.4.1 The normal distribution method

For each of the indices, the normal distribution curve is fitted to the logarithmic price changes using Statgraphics. The optimal margin level for a long position is given by the formula:

$$\pi^{long} = 1 - [1 - 1/(2.\Pi.\sigma^2)^{0.5} \int_{-\infty}^{r^{long}} e^{-[(r-\mu)^2]/2\sigma^2} dr]^n \qquad ...(4)$$

where

π^{long} is the probability of margin violation;
r is the optimal margin level;
μ is the average logarithmic price change for the index;
σ is standard deviation of logarithmic price changes for the index;
n is the period of days to which the probability of margin violation refers (in this case 30 and 60 days).

The optimal margin level for a short position is given by the formula:

$$\pi^{short} = 1 - [1/(2.\Pi.\sigma^2)^{0.5} \int_{-\infty}^{r^{long}} e^{-[(r-\mu)^2]/2\sigma^2} dr]^n \qquad ...(5)$$

3.4.2 The Frechet distribution method

The Frechet extreme value distribution of minimum and maximum price changes over a period of days gives regressed distribution parameters recorded in Table 5. These parameters can be substituted into the formulae given below, derived from Longin (1994: 18), to obtain the margin levels.

For a long position, the optimal margin level, r^{long}, under the Frechet distribution is:

$$r^{long} = \beta^{min} - \{\alpha^{min}.[1 - (- LN(1 - \pi^{long}))^{\tau min}]/\tau^{min}\} \qquad ...(6)$$

For a short position, the optimal margin level, r^{short}, under the Frechet distribution is:

$$r^{short} = \beta^{max} + \{\alpha^{max}.[1 - (- LN(1 - \pi^{short}))^{\tau max}]/\tau^{max}\} \qquad ...(7)$$

These formulae are used to compute margin level, r, given a probability of violation, π.

3.4.3 The empirical margin

The method of calculating the empirical margins, $r^{long}$and r^{short}, was the same as that of Longin (1994:19):

N. π^{long} = the number of MINn such that MINn < r^{long}, for long positions

N. $(1-\pi^{short})$ = the number of MAXn such that MAXn < r^{short}, for short positions

where N is the total number of observed periods of either 30 or 60 days. The other variables are as defined before.

3.4.4 Comparative results

The results of these comparisons are given in Appendices 1 to 12. If Appendix 1 is used as an example, the optimal margin level increases in absolute value as the acceptable margin violation probability decreases: e.g. a margin violation probability of 0,5 requires a margin under normality of -2,12%, but a margin violation probability of 0,001 requires a margin under normality of -4,28%. The daily margin violation level acceptable by SAFEX, 0,0005, translates into a probability of violation of 0,0148918 over 30 days.

Appendix 1 indicates that at the 30 day equivalent of SAFEX's margin violation level, 0,0148918, the margin required by the normal distribution is - 3,53%. The Frechet distribution requires a margin of -11,31% which is very close to the empirical determination of price change of -11,85% at this level of probability. The margining under the normal distribution method is therefore rather inadequate for extreme violation probabilities such as the level chosen by SAFEX. The margin level of -3,53% which would be adequate under normality for the SAFEX violation equivalent of 0,0148918, will only cover a margin violation probability of more than ten times that amount under either Frechet or empirical observation. The normal method of calculating margin under small probabilities of violation therefore severely underestimates the incidence of extreme price changes. Appendices 2 to 12 support these findings. Appendix 3 shows that for the long contract on the GLDI with a SAFEX margin violation equivalent of 0,0148918 for 30 days, the margin under normality is -7,23% where the Frechet and empirical margins called for are -11,49% and -11,02% respectively. Appendix 2 shows that for the INDI with a SAFEX margin violation equivalent of 0,0148918 for 30 days, the margin under normality is -2,91% where the Frechet and empirical margins called for are -12,19% and -12,38% respectively.

As in Longin's study (1994: 21), the Frechet margins for this study of the South African indices are close to the empirical ones, indicating the good fit of the Frechet distribution to the extremes. The normal distribution obviously has a very poor fit to the extremes.

In Appendix 7, which refers to the calculation of margins for long positions on ALSI for the period 1986 to 1998, the SAFEX daily margin violation probability of 0,0005 becomes an equivalent violation probability of 0,0295618 over 60 days. Note that the margins are not exactly the same as for the equivalent 30 day violation probability either empirically or under Frechet, as found in Appendix 1. This is because only the most extreme price change in each 60 day period was considered, and consequently the shape of the distribution alters slightly by virtue of some excluded 30 day extremes. The 60 day interval calculations show the underestimation of extreme price changes by the normal distribution just as clearly as the 30 day interval calculations.

3.5 Stability of optimal margin levels over time

In order to have some comparison with the SAFEX method of calculating margins, which not only relies on the normal distribution, but also on volatility of price changes, margins for the following six particular days on the ALSI were calculated (SAFEX changed their margins on these days): 93/03/15; 93/12/29; 95/02/13; 96/01/08; 96/12/17; 97/12/29. Extreme values over 30 day intervals were calculated. The margins were based on the price changes of the 750 days preceding the margin date. SAFEX bases its volatility calculation on the larger of the volatility of the 750 days preceding the margin to be set, and the implied volatility of options on the futures contract.

In considering optimal margins for a SAFEX probability of margin violation of once in every 2 000 days, margins arrived at based on data gathered over 12 years is, however, preferable to data gathered over a short period of 750 days.

In some cases the margin was not calculated from the Frechet distribution, but the Weibull. This is because the tail index,

τ, was positive. In the instances where Weibull distributions were called for, the margins calculated under normality were much closer to the Weibull and empirical margins than when the Frechet distribution was called for. This is because there is a much larger amount of extreme values under Frechet which are unexplained by the normal distribution. Under the Weibull distribution, the distribution of extremes tapers off rapidly as it does under the normal distribution. The Weibull results were read off using the 'Tail areas probabilities' option in the Statgraphics Weibull distribution menu.

The results for the 750 day data sets showed that the normal distribution was more appropriate for margin setting when a Weibull distribution was called for by the rare instance of a positive tail index. However, the actual margins set by SAFEX in these instances were, as shown in the Table 6, ironically, mostly a lot closer to the Frechet margins calculated for the entire 12 year period.

Table 6 tabulates the Rand equivalent of the percentage logarithmic price changes compared with SAFEX's actual set margin. The margin was calculated using the futures index price for the day, the expected price change at SAFEX's risk parameter, and logarithmic price changes.

SAFEX margins are reasonably close to the short contract margins of the Frechet distribution, notably under the ALSI and INDI contracts. However, the long ALSI and INDI contracts are under margined by SAFEX for the twelve year period. The consideration that there are more large index drops than gains seem to be ignored by SAFEX in their absence of long/short margin differentiation. The Frechet distribution makes this differentiation simply and effectively.

In SAFEX's margin calculations, the implied volatility of the normal distribution found from at the money options on the futures is, after all, based on expectation, not on fact. Ironically, SAFEX's actual margins are more closely correlated to the Frechet 12 year margin determinations than the 750 day determinations. The 12 year analyses are a more accurate reflection of what price changes are likely under remote margin violation possibilities such as 0,0005.

Edwards and Neftci (1988:639) point out that only the volatility due to extreme price changes over the long term is relevant. The 750 day analyses (30 days times 25) do not allow an accurate calculation of margin under this small chance (1 in 25 of 30 day intervals is only 0,04). Volatilities of less extreme price changes will vary significantly over the shorter term (750 days), but do not affect the margin calculation process which is concerned with extreme price changes at probabilities of one in 2 000 in SAFEX's case.

It appears that the Frechet 12 year distribution method of calculating margins at a daily probability of violation of 0,0005 is simpler and more effective than a method based on normality and volatility of shorter time periods. Price changes do not follow a normal distribution, and volatilities over short time periods are not effective in predicting remote probabilities of violation.

Table 6: Margin comparisons and calculations in Rand terms

Contract type

Period of observed price change	*Date	Margin under Frechet	Margin under Weibull	Empirical Margin	Margin under Normality	** Safex margin	
ALSI		930315					
short	1986-98		2161		2088	1279	2000
long	1986-98		3699		3865	1200	2000
short	750			922	972	1050	2000

	day						
long	750 day		3323		1835	1016	2000
ALSI		931229					
short	1986-98		3121		3015	1846	2750
long	1986-98		5342		5582	1732	2750
short	750 day		1914		1645	1413	2750
long	750 day		3134		2616	1297	2750
ALSI		950213					
short	1986-98		3324		3211	1967	3000
long	1986-98		5689		5945	1845	3000
short	750 day		2516		1939	1593	3000
long	750 day			1629	1608	1495	3000
ALSI		960108					
short	1986-98		4160		4019	2462	3250
long	1986-98		7121		7441	2310	3250
short	750 day		3253		2427	1891	3250
long	750 day			2058	1865	1722	3250
ALSI		961217					

short	1986-98		3642		3518	2155	3500
long	1986-98		6235		6515	2022	3500
short	750 day		3149		2125	1554	3500
long	750 day			1808	1632	1462	3500

(based on probability of margin violation over 30 days of 0,0148918, which is equivalent to a margin violation probability of 0,0005 for one day - the SAFEX risk parameter)

* Dates at which the SAFEX margins were changed to the levels indicated.

** Safex margin is based on the higher of the 750 day volatility and the overnight volatility implied by at-the-money options quoted on the futures. 'Period of observed price change' does therefore not refer to 'Safex margin'.

Table 6 (continued)

Contract type	Period of observed price change	*Date	Margin under Frechet	Margin under Weibull	Empirical Margin	Margin under Normality	** Safex margin
ALSI		971229					
short	1986-98		3353		3239	1984	4500
long	1986-98		5739		5998	1862	4500
short	750 day		4949		3719	1612	4500
long	750 day		7044		5998	1555	4500
GLDI		930315					
short	1986-98		1285		1170	787	1000

long	1986-98		1143		1099	734	1000
GLDI		931229					
short	1986-98		2653		2418	1626	3000
long	1986-98		2361		2270	1517	3000
GLDI		950213					
short	1986-98		2025		1845	1241	2500
long	1986-98		1802		1732	1158	2500
GLDI		960108					
short	1986-98		1897		1728	1163	2500
long	1986-98		1688		1623	1085	2500
GLDI		961217					
short	1986-98		1239		1129	760	1500
long	1986-98		1103		1060	709	1500
GLDI		971229					
short	1986-98		762		695	467	1000
long	1986-98		678		652	436	1000

INDI		930315					
short	1986-98		2800		2800	1383	2000
long	1986-98		5144		5219	1285	2000
INDI		931229					
short	1986-98		3580		3580	1769	2750
long	1986-98		6576		6672	1643	2750
INDI		950213					
short	1986-98		4085		4085	2018	3000
long	1986-98		7503		7613	1875	3000
INDI		960108					
short	1986-98		5273		5273	2605	3750
long	1986-98		9687		9829	2421	3750
INDI		961217					
short	1986-98		4706		4706	2325	4500
long	1986-98		8644		8771	2160	4500
INDI		971229					
short	1986-98		4228		4228	2089	5500

long	1986-98		7767		7881	1941	5500

**Source of SAFEX margins: SAFEX Administrative Office

4. Conclusions

The research indicates that Longin's method is a more efficient and sounder way of calculating optimal margins on futures contracts on the ALSI, INDI, and GLDI. It was firstly shown that the normal distribution poorly reflects the empirical distribution of daily log price changes on ALSI, INDI, and GLDI over the long term (12 years). This is especially the case for extreme price changes - those occurring at the low probabilities such as the SAFEX daily violation probability of 0,0005. The Chi-square tests conducted showed that the normal distribution clearly under-predicted the incidence of empirical extreme price changes. Low probability price changes (extreme price changes) were clearly described using an extreme value distribution, in these cases a Frechet distribution.

The setting of margins for ALSI, INDI, and GLDI showed that the Frechet margins for the 12 year period 1986 to 1998 were much closer to margins suggested by empirically observed price changes than the normal margins were. This finding was especially apparent at low probabilities of violation (for extreme price changes) such as the SAFEX level of 0,0005.

It can therefore be concluded that an extreme value distribution is clearly better than the normal distribution in reflecting price changes on the above indices. Once the incidence of low probability price changes are more accurately predicted by an extreme value distribution, more accurate margins can be set from that information.

SAFEX does not use a simple normal distribution method of calculating margins. Even though SAFEX uses market volatility to alter the margin levels, the SAFEX margins are ironically much closer to the margin levels based on 12 year volatility of the Frechet extreme value distribution than margins calculated by the normal method based on a 750 day volatility.

There is a greater incidence of price drops on the indices which SAFEX does not cater for in their margining of long contracts.

It can be concluded that the Frechet method is more efficient and accurate than the SAFEX method of calculating margins because:

· The Frechet distribution only deals with extreme price changes, the most extreme changes every 30 days. It only considers volatility due to large price variation, therefore its fit with extreme changes at low probabilities is superior. The SAFEX margin violation probability level of once in 2 000 days does not justify setting margins on short term volatility, volatility which furthermore does not restrict itself to extreme changes.
· The Frechet method differentiates between margins for long and short contracts. The fact that the margin called for under long contracts are sometimes almost double those of short contracts is ignored by SAFEX.

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Appendix 1: Comparisons of percentage logarithmic margins obtained for long positions on the ALSI, under various margin violation probabilities, with a violation interval of 30 days, over the period February 1986 to February 1998

AL30MI

Margin violation probability	Margin under Frechet	Empirical margin	Margin under normality
0,5	-1,67	-1,75	-2,12

Appendix 2: Comparisons of percentage logarithmic margins obtained for long positions on the INDI, under various margin violation probabilities, with a violation interval of 30 days, over the time period February 1986 to February 1998

IN30MI

Margin violation probability	Margin under Frechet	Empirical margin	Margin under normality
0,5	-1,20	-1,20	-1,74

0,25	-2,64	-2,46	-2,50
0,1	-4,37	-3,93	-2,88
0,05	-6,23	-6,80	-3,13
0,04	-6,96	-8,46	-3,21
0,03	-8,03	-11,20	-3,30
0,02	-9,80	-11,85	-3,43
0,0148918	-11,31	-11,85	-3,53
0,01	-13,72	-12,52	-3,65
0,005	-19,16	~	-3,85
0,001	-41,39	~	-4,28

Margin level %	Probability of violation		
	Under Frechet	Empirically	Under Normality
-1	0,831	0,850	0,996
-2	0,389	0,400	0,593
-3	0,200	0,150	0,072
-4	0,118	0,090	0,003
-5	0,077	0,070	0,000
-10	0,019	0,030	0,000
-15	0,008	0,000	0,000
-20	0,005	0,000	0,000
-30	0,002	0,000	0,000

Note: The margin violation probability of 0,0148918 is the 30 day equivalent of SAFEX's daily violation parameter of 0,0005.

0,25	-2,09	-2,03	-2,05
0,1	-3,85	-3,02	-2,37
0,05	-5,92	-6,75	-2,58
0,04	-6,77	-7,73	-2,64
0,03	-8,05	-11,75	-2,72
0,02	-10,24	-12,38	-2,83
0,0148918	-12,19	-12,38	-2,91
0,01	-15,40	-13,35	-3,01
0,005	-23,07	~	-3,18
0,001	-58,67	~	-3,54

Margin level %	Probability of violation		
	Under Frechet	Empirically	Under Normality
-1	0,597	0,590	0,978
-2	0,266	0,250	0,285
-3	0,147	0,100	0,010
-4	0,094	0,070	0,000
-5	0,066	0,050	0,000
-10	0,021	0,030	0,000
-15	0,010	0,000	0,000
-20	0,006	0,000	0,000
-30	0,003	0,000	0,000

Note: The margin violation probability of 0,0148918 is the 30 day equivalent of SAFEX's daily violation parameter of 0,0005.

Appendix 3: Comparisons of percentage logarithmic margins obtained for long positions on the GLDI, under various margin violation probabilities, with a violation interval of 30 days, over the period February 1986 to February 1998

GL30MI

Margin violation probability	Margin under Frechet	Empirical margin	Margin under normality
0,5	-3,83	-3,91	-4,40
0,25	-5,13	-5,12	-5,15
0,1	-6,90	-7,20	-5,93
0,05	-8,40	-8,49	-6,44
0,04	-8,92	-8,84	-6,59
0,03	-9,62	-9,68	-6,78
0,02	-10,68	-11,02	-7,05
0,0148918	-11,49	-11,02	-7,23
0,01	-12,67	-12,94	-7,48
0,005	-14,94	~	-7,88
0,001	-21,52	~	-8,76

Margin level %	Probability of violation		
	Under Frechet	Empirically	Under Normality
-1	1,000	1,000	1,000
-2	0,964	0,970	0,998
-3	0,736	0,730	0,934
-4	0,458	0,450	0,652
-5	0,268	0,260	0,293
-10	0,026	0,020	0,000
-15	0,005	0,000	0,000
-20	0,001	0,000	0,000

Appendix 4: Comparisons of percentage logarithmic margins obtained for short positions on the ALSI, under various margin violation probabilities, with a violation interval of 30 days, over the period February 1986 to February 1998

AL30MA

Margin violation probability	Margin under Frechet	Empirical margin	Margin under normality
0,5	1,78	1,81	2,22
0,25	2,47	2,48	2,60
0,1	3,44	3,11	2,98
0,05	4,28	3,58	3,23
0,04	4,57	3,59	3,31
0,03	4,97	4,74	3,41
0,02	5,58	5,86	3,54
0,0148918	6,06	5,86	3,63
0,01	6,76	6,54	3,75
0,005	8,13	~	3,95
0,001	12,25	~	4,39

Margin level %	Probability of violation		
	Under Frechet	Empirically	Under Normality
1	0,916	0,890	0,998
2	0,402	0,430	0,673
3	0,149	0,140	0,096
4	0,062	0,040	0,004
5	0,029	0,030	0,000
10	0,002	0,000	0,000
15	0,000	0,000	0,000
20	0,000	0,000	0,000

-30	0,000	0,000	0,000

Note: The margin violation probability of 0,0148918 is the 30 day equivalent of SAFEX's daily violation parameter of 0,0005.

30	0,000	0,000	0,000

Note: The margin violation probability of 0,0148918 is the 30 day equivalent of SAFEX's daily violation parameter of 0,0005.

Appendix 5: Comparisons of percentage logarithmic margins obtained for short positions on the INDI, under various margin violation probabilities, with a violation interval of 30 days, over the period February 1986 to February 1998

IN30MA

Margin violation probability	Margin under Frechet	Empirical margin	Margin under normality
0,5	1,41	1,47	1,87
0,25	1,99	1,90	2,18
0,1	2,92	2,55	2,50
0,05	3,83	3,35	2,71
0,04	4,18	3,68	2,77
0,03	4,66	4,07	2,85
0,02	5,42	6,06	2,96
0,0148918	6,06	6,06	3,04
0,01	7,02	6,56	3,14
0,005	9,06	~	3,30
0,001	16,32	~	3,67

Margin level %	Probability of violation		
	Under Frechet	Empirically	Under Normality
1	0,802	0,810	0,992
2	0,248	0,230	0,385
3	0,094	0,060	0,017

Appendix 6: Comparisons of percentage logarithmic margins obtained for short positions on the GLDI, under various margin violation probabilities, with a violation interval of 30 days, over the period February 1986 to February 1998

GL30MA

Margin violation probability	Margin under Frechet	Empirical margin	Margin under normality
0,5	4,32	4,45	4,38
0,25	5,79	5,63	5,14
0,1	7,59	6,78	5,91
0,05	8,97	8,49	6,42
0,04	9,42	8,64	6,57
0,03	10,01	10,25	6,77
0,02	10,87	10,54	7,03
0,0148918	11,51	10,54	7,21
0,01	12,39	11,16	7,46
0,005	13,99	~	7,87
0,001	18,05	~	8,75

Margin level %	Probability of violation		
	Under Frechet	Empirically	Under Normality
1	0,999	1,000	1,000
2	0,960	0,950	0,997
3	0,803	0,820	0,931

4	0,045	0,040	0,000
5	0,025	0,030	0,000
10	0,004	0,000	0,000
15	0,000	0,000	0,000
20	0,001	0,000	0,000
30	0,000	0,000	0,000

Note: The margin violation probability of 0,0148918 is the 30 day equivalent of SAFEX's daily violation parameter of 0,0005.

4	0,572	0,570	0,645
5	0,367	0,400	0,288
10	0,030	0,040	0,000
15	0,003	0,000	0,000
20	0,000	0,000	0,000
30	0,000	0,000	0,000

Note: The margin violation probability of 0,0148918 is the 30 day equivalent of SAFEX's daily violation parameter of 0,0005.

Appendix 7: Comparisons of percentage logarithmic margins obtained for long positions on the ALSI, under various margin violation probabilities, with a violation interval of 60 days, over the period February 1986 to February 1998

AL60MI

Margin violation probability	Margin under Frechet	Empirical margin	Margin under normality
0,5	-2,27	-2,27	-2,42
0,25	-3,60	-3,27	-2,76
0,1	-6,25	-6,80	-3,12
0,05	-9,37	-11,53	-3,36
0,04	-10,67	-11,85	-3,43
0,03	-12,60	-12,19	-3,52
0,0295618	-12,71	-12,19	-3,53
0,2	-15,94	-12,52	-3,64
0,01	-23,80	~	-3,85
0,005	-35,56	~	-4,04
0,001	-90,48	~	-4,46

Margin	Probability of violation

Appendix 8: Comparisons of percentage logarithmic margins obtained for long positions on the INDI, under various margin violation probabilities, with a violation interval of 60 days, over the period February 1986 to February 1998

IN60MI

Margin violation probability	Margin under Frechet	Empirical margin	Margin under normality
0,5	-1,70	-1,70	-1,99
0,25	-3,02	-2,79	-2,28
0,1	-5,87	-6,75	-2,57
0,05	-9,46	-12,07	-2,77
0,04	-11,01	-12,38	-2,83
0,03	-13,38	-12,87	-2,90
0,02	-13,51	-12,87	-2,91
0,0148918	-17,58	-13,35	-3,01
0,01	-27,99	~	-3,17
0,005	-44,49	~	-3,34
0,001	-130,30	~	-3,68

Margin	Probability of violation

level %	Under Frechet	Empirically	Under Normality
-1	0,979	0,960	1,000
-2	0,587	0,660	0,835
-3	0,333	0,260	0,139
-4	0,211	0,180	0,006
-5	0,146	0,140	0,000
-10	0,045	0,060	0,000
-15	0,022	0,000	0,000
-20	0,014	0,000	0,000
-30	0,007	0,000	0,000

Note: The margin violation probability of 0,0295618 is the 60 day equivalent of SAFEX's daily violation parameter of 0,0005.

level %	Under Frechet	Empirically	Under Normality
-1	0,794	0,800	1,000
-2	0,418	0,420	0,489
-3	0,253	0,180	0,021
-4	0,172	0,140	0,000
-5	0,126	0,120	0,000
-10	0,046	0,060	0,000
-15	0,025	0,000	0,000
-20	0,017	0,000	0,000
-30	0,009	0,000	0,000

Note: The margin violation probability of 0,0295618 is the 60 day equivalent of SAFEX's daily violation parameter of 0,0005.

Appendix 9: Comparisons of percentage logarithmic margins obtained for long positions on the GLDI, under various margin violation probabilities, with a violation interval of 60 days, over the period February 1986 to February 1998

GL60MI

Margin violation probability	Margin under Frechet	Empirical margin	Margin under normality
0,5	-4,63	-4,82	-5,00
0,25	-6,19	-5,98	-5,70
0,1	-8,32	-8,49	-6,42
0,05	-10,10	-10,35	-6,90
0,04	-10,72	-11,02	-7,04
0,03	-11,55	-11,98	-7,22
0,0295618	-11,60	-11,98	-7,23
0,2	-12,80	-12,94	-7,47
0,01	-15,14	~	-7,88

Appendix 10: Comparisons of percentage logarithmic margins obtained for short positions on the ALSI, under various margin violation probabilities, with a violation interval of 60 days, over the period February 1986 to February 1998

AL60MA

Margin violation probability	Margin under Frechet	Empirical margin	Margin under normality
0,5	2,11	2,15	2,52
0,25	2,93	2,78	2,87
0,1	4,17	3,58	3,23
0,05	5,33	5,30	3,46
0,04	5,76	5,86	3,53
0,03	6,35	6,20	3,62
0,0295618	6,38	6,20	3,63
0,2	7,28	6,54	3,75
0,01	9,14	~	3,95

0,005	-17,80	~	-8,27
0,001	-25,44	~	-9,12

Margin level %	Probability of violation		
	Under Frechet	Empirically	Under Normality
-1	1,000	1,000	1,000
-2	0,991	1,000	1,000
-3	0,876	0,860	0,996
-4	0,645	0,640	0,879
-5	0,427	0,440	0,500
-10	0,052	0,040	0,000
-15	0,010	0,000	0,000
-20	0,003	0,000	0,000
-30	0,000	0,000	0,000

Note: The margin violation probability of 0,0295618 is the 60 day equivalent of SAFEX's daily violation parameter of 0,0005.

Appendix 11: Comparisons of percentage logarithmic margins obtained for short positions on the INDI, under various margin violation probabilities, with a violation interval of 60 days, over the period February 1986 to February 1998

IN60MA

Margin violation probability	Margin under Frechet	Empirical margin	Margin under normality
0,5	1,70	1,69	2,12
0,25	2,42	2,32	2,40
0,1	3,72	2,98	2,70
0,05	5,12	5,07	2,90

0,005	11,45	~	4,14
0,001	19,13	~	4,56

Margin level %	Probability of violation		
	Under Frechet	Empirically	Under Normality
1	0,984	0,960	1,000
-2	0,550	0,580	0,893
3	0,235	0,240	0,182
4	0,112	0,080	0,008
5	0,060	0,060	0,000
10	0,008	0,000	0,000
15	0,002	0,000	0,000
20	0,001	0,000	0,000
30	0,000	0,000	0,000

Note: The margin violation probability of 0,0295618 is the 60 day equivalent of SAFEX's daily violation parameter of 0,0005.

Appendix 12: Comparisons of percentage logarithmic margins obtained for short positions on the GLDI, under various margin violation probabilities, with a violation interval of 60 days, over the period February 1986 to February 1998

GL60MA

Margin violation probability	Margin under Frechet	Empirical margin	Margin under normality
0,5	5,09	5,36	4,98
0,25	6,71	6,43	5,68
0,1	8,79	8,49	6,40
0,05	10,44	10,40	6,88

0,04	5,67	6,06	2,96
0,03	6,48	6,31	3,03
0,0295618	6,52	6,31	3,04
0,2	7,82	6,56	3,14
0,01	10,80	~	3,30
0,005	14,95	~	3,46
0,001	32,00	~	3,81

Margin level %	Probability of violation		
	Under Frechet	Empirically	Under Normality
1	0,952	0,940	1,000
2	0,368	0,340	0,622
3	0,159	0,100	0,034
4	0,085	0,080	0,000
5	0,053	0,060	0,000
10	0,012	0,000	0,000
15	0,005	0,000	0,000
20	0,003	0,000	0,000
30	0,001	0,000	0,000

Note: The margin violation probability of 0,0295618 is the 60 day equivalent of SAFEX's daily violation parameter of 0,0005.

0,04	11,00	10,54	7,02
0,03	11,74	10,85	7,21
0,0295618	11,78	10,85	7,21
0,2	12,82	11,16	7,46
0,01	14,79	~	7,86
0,005	16,93	~	8,25
0,001	22,67	~	9,10

Margin level %	Probability of violation		
	Under Frechet	Empirically	Under Normality
1	1,000	1,000	1,000
2	0,994	0,980	1,000
3	0,921	0,960	0,995
4	0,734	0,700	0,874
5	0,517	0,560	0,500
10	0,060	0,080	0,000
15	0,009	0,000	0,000
20	0,002	0,000	0,000
30	0,000	0,000	0,000

Note: The margin violation probability of 0,0295618 is the 60 day equivalent of SAFEX's daily violation parameter of 0,0005.