The Investment Analysts Journal

Nommer 18 Oktober 1981

Die Beleggings-Navorsers Tydskrif

Number 18 October 1981

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Inhoud

This issue in brief

Business investment decisions under inflation: Some South African evidence

Is money illusion a factor which affects the performance of the real economy? This is a question likely to provoke disagreement between different schools of thought regarding market efficiency and the rationality of expectations. In this article by Robert Reeve, Senior Lecturer in the Department of Accountancy at the University of Natal, Durban, the results of a study concerned with the effect of inflation on investment in South Africa are reported and evaluated. The study is based on a similar study undertaken in the UK by Carsberg and Hope during the early 1970s and a comparison of the results of each is made. Mr Reeve's findings support the contention that investment appraisal practices in the Republic are biased towards underinvestment in inflationary times.

Analysing the yield curve: A new approach

The yield curve has an entrenched place in the analysis of fixed income security returns. However, its nature and value as an investment tool are not often properly understood. If the curve reflects the outlook of investors of future yields rather than the comparative supply functions of stocks with different terms to maturity, the nature of the expectations concerned have still to be appreciated. Stanley Diller, in a very comprehensive paper, argues that real world investors are more concerned with the near-term future than with the medium or long-term future when making their comparisons. This goes a long way to explaining why yield curves differ as between sectors and how tax considerations affect fixed income evaluation.

Review of the gold sales policy of the United States Treasury (1978-9)

As the largest single holder of gold bullion, the US Treasury is, clearly, in a special position to influence the gold price if this is, indeed, possible and its intention. For this reason, if for no other, the review of the US gold sales policy during the 1978-9 period, by Professor Roger Gidlow of the Department of Business Economics at the University of the Witwatersrand, is important. Professor Gidlow starts with an examination of the reasons for the gold sales and then proceeds to examine their impact on the gold price and the monetary status of the metal. Interestingly, he draws attention to the fact that while the gold sales had the short-run effect of weakening the gold price, their longer-run effect was to accelerate the trend towards reserve diversification out of the dollar. This is one of the reasons for his concluding that a change in Treasury policy could materialise in the near-term.

An illustrative method for valuing annuities

Annuities have an important place in both the theory and practice of finance. In this technical article from the Graduate School of Business in Melbourne, expressions are derived for the valuation of the four main types of annuities, namely, ordinary annuities, annuities due, deferred annuities and perpetuities. In addition, a graphical method for calculating the value of an annuity, that is easy to use, is presented in the form of a nomogram. This illustrates what happens to the value of an annuity when it is subject to changes in the values of imput parameters.

The use of comprehensive cash flow analysis as a basis for risk measurement and determination of debit capacity.

In his last Investment Basics article (see IAJ April 1981) Mr A C Valsamakis discussed the 'return of investment' concept showing how it could be used as a tool in financial management. In this article he develops his theme further to include an investigation of the matter of gearing and its specific effect on both return and risk. Particular emphasis is placed on cash flow evaluation and its importance to the survival of the firm.

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The Investment Analysts Journal

Eighteenth issue

October 1981

It has never been a policy of this Journal to express an opinion about the future course of stock-market share prices or even about that of the general economy, either in editorials or in the columns of our contributors. Prognostications, both financial and economic, have a habit of quickly becoming outdated and are a sure road to academic irrelevance. Certainly, they diminish the durability of analysis and research, and durability is a quality upon which we, of the Journal, place a high value in the selection of our content. Nevertheless, we take this opportunity to express our interest in the fact that the JSE Actuaries Industrial Index has, within the last few weeks, reached a new high almost simultaneously with the publication by the South African Reserve Bank of a seasonally adjusted per annum estimate for the deficit on the current account of the balance of payments of approximately R3 000 m for the first six months of the year. No observer of money and capital markets in South Africa can be unmoved by such an apparent contradiction, especially having regard to the fact that, in the past, industrial share prices and the BoP deficit have tended to move in an inverse relationship.

Money and capital markets are not always right in terms of ex post assessment; but as barometers of expectations, they need to be taken seriously, especially when there exists a fair accumulation of evidence as to their efficiency. In present circumstances the rise in share prices is telling us something. The important thing to decide is "what?" Our guess is that it has at least something to do with the renewed fall in short-term interest rates, with the recent recovery in the gold price, with the still strong flow of good profits results, with the fact that dividend and earnings yields never did decline to unreasonable levels during the erstwhile economic upswing, and with the weight of new institutional funds seeking investment while scrip has remained essentially in short supply.

However, while these reasons may seem plausible, they do not probe far enough below the surface to provide a really adequate explanation. Why have interest rates fallen again? What is behind the increase in the gold price? These are the questions investment analysts need to be asking now if they want to find the answers to the future movements of share prices.

The fall in South African domestic interest rates can be simply explained but not easily understood given the Government's declared objective of reducing the level of inflation in the country and its looming BoP deficit. When off-shore borrowing is made deliberately attractive, as it was during August by maniplulation of the forward cover mechanism, and when the rand is devalued by 22% and then kept down despite a recovery in the gold price, the

Die Beleggingsnavorsers Tydskrif

Agtiende uitgawe

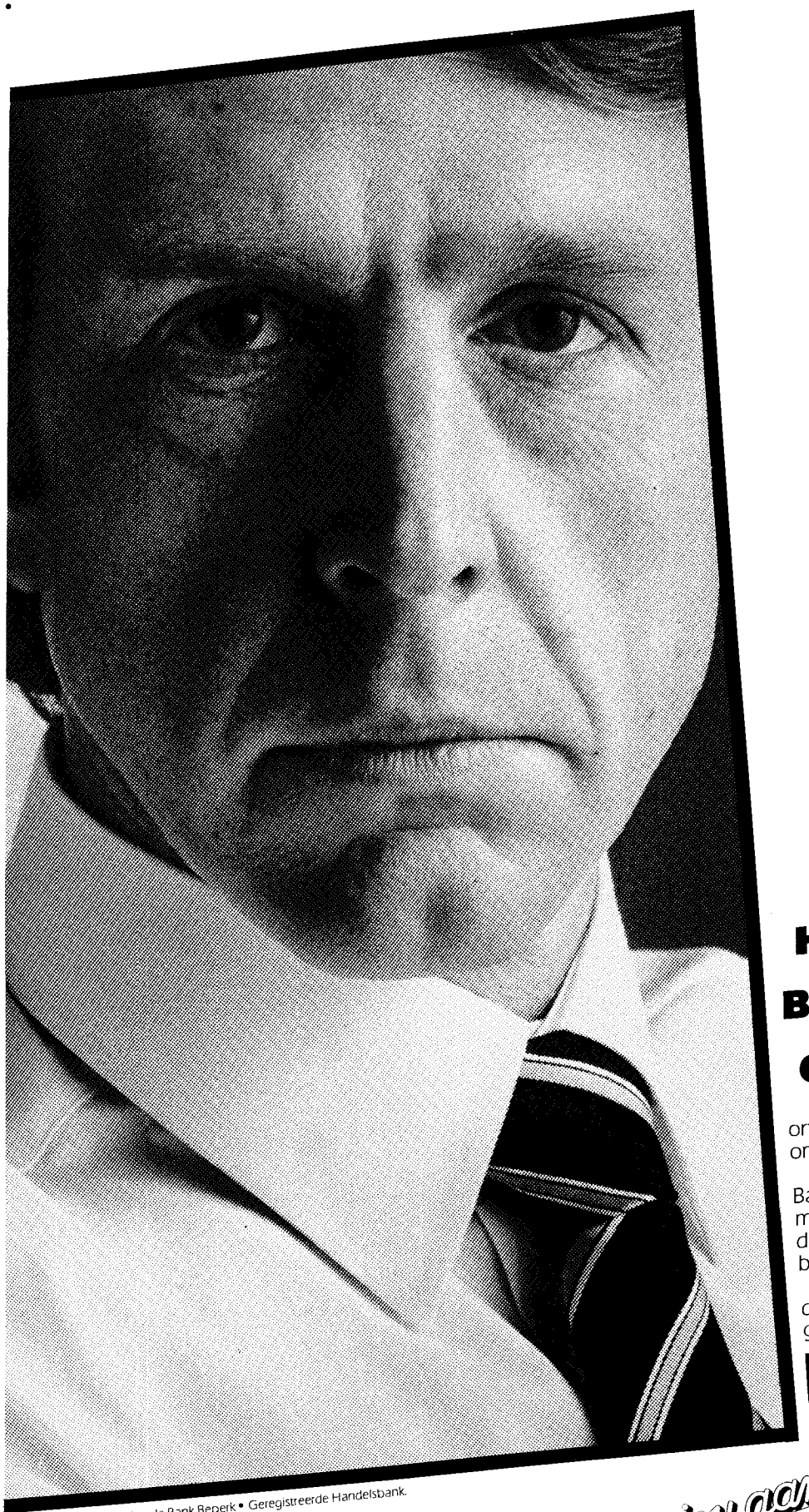
Oktober 1981

Dit was nog nooit hierdie Tydskrif se beleid om 'n mening uit te spreek oor die toekomstige verloop van effektemarkaandelepryse of selfs oor die verloop van die ekonomie in die algemeen nie, hetsy in redaksionele artikels of in die rubrieke van ons bydraers. Voorspellings, finansieel sowel as ekonomies, het die manier om baie gou verouderd te raak en is gedoem tot akademiese ontoepaslikheid. Dié voorspellings doen beslis afbreuk aan die bestendigheid van ontleding en navorsing, en bestendigheid is 'n eienskap waaraan ons by die Tydskrif groot waarde heg wanneer dit kom by die selektering van ons inhoud. Nietemin maak ons van dié geleentheid gebruik om ons belangstelling uit te spreek in die feit dat die JE Aktuariële Nywerheidsindeks binne die afgelope paar weke 'n nuwe hoogtepunt bereik het, byna gelyktydig met die verskyning van 'n jaarlikse raming, seisoensinvloede uitgeskakel, vir die tekort op die lopende rekening van die betalingsbalans van ongeveer R3 000 m vir die eerste ses maande van die jaar deur die Suid-Afrikaanse Reserwebank. Geen waarnemer van geld- en kapitaalmarkte in Suid-Afrika kan só 'n duidelike teenstrydigheid miskyk nie, veral met die oog op die feit dat nywerheidsaandelepryse en die betalingsbalans in die verlede geneig het om in 'n omgekeerde verhouding tot mekaar te staan.

Geld- en kapitaalmarkte het nie altyd gelyk wanneer dit kom by 'n skatting ex post nie, maar as barometers van verwagtinge moet hulle ernstig opgeneem word, veral wanneer daar redelike bewyse van hulle doeltreffendheid bestaan. In die huidige omstandighede hou die styging in aandelepryse vir ons 'n boodskap in. Die belangrike vraag is net "wat?" Ons raaiskoot is dat dit minstens iets te doen het met die hernieude daling in korttermynrentekoerse, met die onlangse herstel in die goudprys, met die steeds sterk beweging van goeie winsresultate, met die feit dat dividend- en winsopbrengste gedurende die onlangse ekonomiese opswaai nooit na onredelike peile afgeneem het nie, en met die gewig van nuwe institusionele fondse wat vir belegging beskikbaar is terwyl daar in hoofsaak 'n onvoldoende aanbod van effekte was.

Hoewel dié redes aanvaarbaar mag klink, is hulle nie grondig genoeg om 'n werklik toereikende verduideliking te bied nie. Waarom het rentekoerse weer eens gedaal? Wat het aanleiding gegee tot die styging in die goudprys? Dit is die vrae wat beleggingsnavorsers nou moet stel as hulle die antwoorde op die toekomstige bewegings in aandelepryse wil vind.

Die daling in die Suid-Afrikaanse binnelandse rentekoerse kan maklik verklaar, maar nie so maklik verstaan word nie, gesien die Regering se verklaarde doelstelling om die inflasiepeil in die land en sy opdoemende betalings-



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cash flow to the money market must per force be increased. The trouble is that, when that happens, the credibility of anti-inflationary policies must also be brought into question. Such behaviour on the part of the monetary authorities must suggest that policy objectives are different from what they have been declared to be or have changed because the foreign exchange reserves have suddenly become a more important problem than either the problem of excessive money supply growth or the problem of resurgent inflation.

But is it right to separate the balance of payments problem from the inflation problem in our present policy formulation? It is right to believe that by taking the easy option and avoiding an end-August money crunch, a cash problem of inadequate foreign exchange reserves can be solved with any degree of permanence? We think not. On the contrary, we would strongly argue that by causing short-term money market trading rates in South Africa to fall again, to levels significantly below either comparable interest rates overseas or local inflation, and by devaluing the rand, the BoP problem will actually be made worse. The logic behind this kind of argument has to do with the fact that neither the demand for our exports not the total demand for our imports are price elastic. Export demand will only really recover when growth in the economies of the major industrial countries recovers. Import demand will only really be discouraged when local business expectations of domestic volume sales and inflation are reduced. There are, too, imports in the pipeline that cannot be headed off by any kind of exchange manipulation. The devaluation of the rand, thus, merely means that we will be sacrificing more scarce foreign exchange per rand when paying for them than will be necessary.

There is no excape from the problem of excess money creation and inflation. It has to be tackled at its roots, not papered over with measures which are evidently expedient but of questionable longer-term value. A still further recovery in the gold price could let us off the short-term cash flow hook we now find ourselves on, but is that something upon which we should rely in our domestic monetary management?

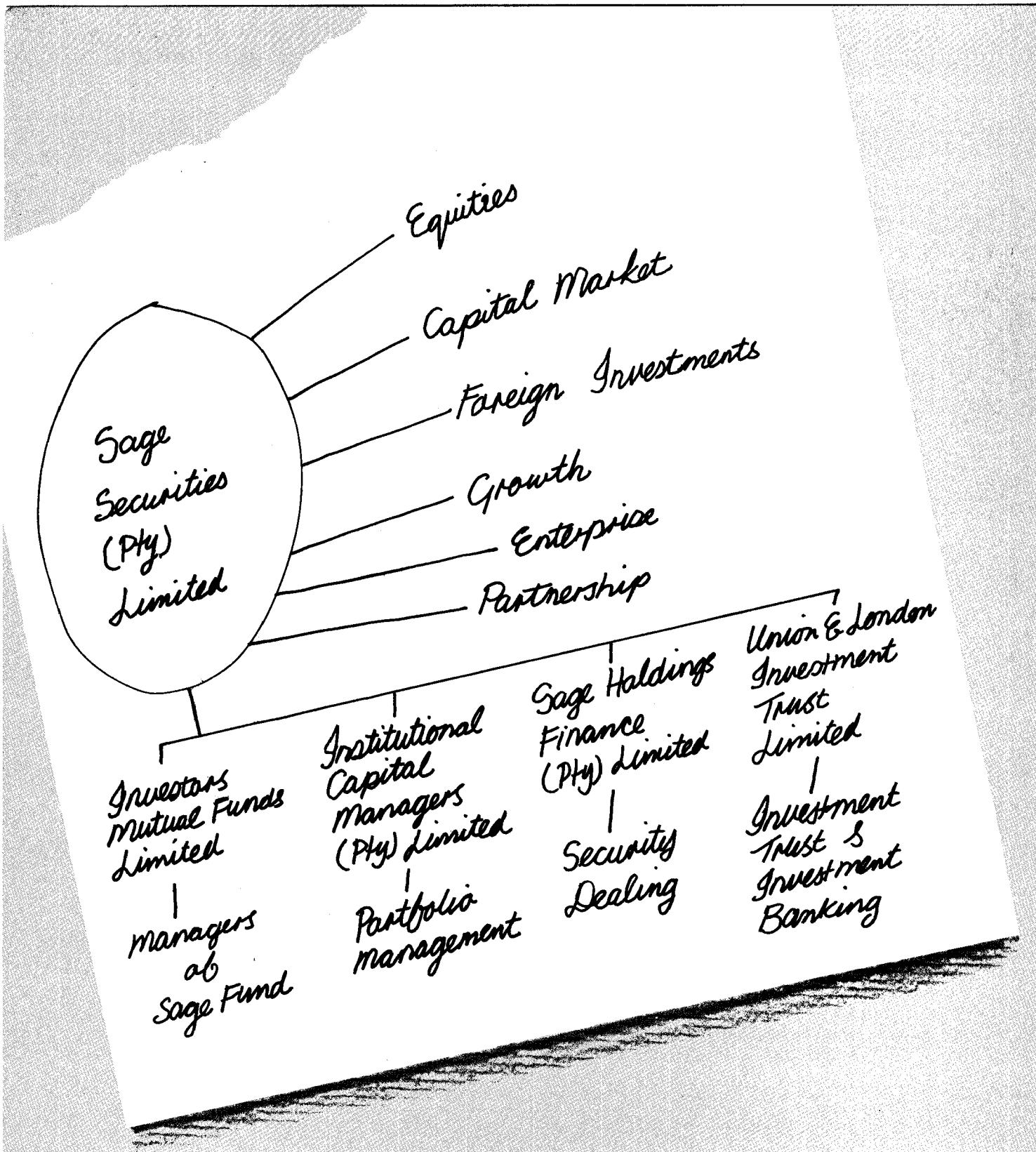
The editor

balanstekort te verlaag. Wanneer buitelandse lenings doelbewus aantreklik gemaak word, soos in Augustus die geval was deur middel van die manipulasie van die termyndekkingsmeganisme, en wanneer die rand met 22% gedevalueer word en dan laag gehou word ten spyte van 'n herstel in die goudprys, moet die kontantvloei na die geldmark noodgedwonge toeneem. Die probleem is dat, wanneer dit gebeur, die geloofwaardigheid van 'n anti-inflasionistiese beleid ook bevraagteken moet word. Dié optrede van die kant van die monetêre owerheid kan net daarop dui dat beleidsdoelstellings nie is wat hulle wil voorgee om te wees nie of dat hulle verander het omdat die buitelandse valuta reserwes skielik 'n belangriker probleem as òf die probleem van oormatige geldvoorraadgroei òf die probleem van hernieude inflasie geword het.

Maar is dit reg om die betalingsbalansprobleem van die inflasieprobleem in ons huidige beleidsformulering te skei? Is dit reg om te glo dat deur die maklike uitweg te kies en 'n geldskaarste aan die einde van Augustus te vermy, 'n kontantprobleem van ontoereikende buitelandse valuta reserwes met enigsins blywende gevolge opgelos kan word? Ons dink nie so nie. Inteendeel, ons voer sterk aan dat deur te veroorsaak dat kortter-myngeldmarkhandelskoerse in Suid-Afrika weer daal na peile wat heelwat laer is as òf vergelykbare rentekoerse oorsee òf plaaslike inflasie, en deur die rand te devalueer, die betalingsbalansprobleem in werklikheid vererger sal word. Die logika wat onderliggend is aan dié soort betoog hou verband met die feit dat nòg die vraag na ons uitvoere nòg die totale vraag na ons invoere pryselasties is. Die uitvoervraag sal eers werklik herstel wanneer groei in die ekonomieë van die groot nywerheidslande herstel. Die invoervraag sal eers werklik ontmoedig word wanneer plaaslike besigheidsverwagtinge van groot binnelandse verkope en inflasie verminder word. Daar is ook invoere wat reeds aan die gang is wat hoegenaamd nie deur enige manipulasie van die buitelandse valuta reserwes gestuit kan word nie. Die devaluasie van die rand beteken dus eenvoudig dat ons meer skaars buitelandse valuta per rand sal moet prysgee wanneer ons daarvoor betaal as wat nodig sal wees.

Die probleem van die skepping van oortollige geld en inflasie kan eenvoudig nie ontduik word nie. Daar moet tot die kern van die probleem deurgedring word en dit moenie omseil word met maatreëls wat klaarblyklik doelmatig, maar van twyfelagtige langtermynwaarde is nie. 'n Nog verdere herstel in die goudprys kan ons red uit die korttermynkontantvloeipenarie waarin ons ons tans bevind, maar kan ons dit bekostig om daarop staat te maak wanneer dit kom by ons binnelandse monetêre bestuur?

Die redakteur



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By Robert Reeve

Business investment decisions under inflation: some South African evidence

*Senior lecturer, Dept. of Accountancy, University of Natal, Durban

1 ABSTRACT

South Africa has experienced double-digit inflation since 1973 which should have an influence on business investment decisions.

The monograph 'Business Investment Decisions Under Inflation' by Bryan Carsberg and Tony Hope which contains the results of a field study carried out in the UK in 1973 is outlined.

The results of a similar field study among major South African industrial companies in 1978 is then described. These results show a remarkable similarity to those obtained in the UK.

It is considered that these results give strong support to the hypothesis that the investment appraisal methods (and particularly the way in which inflation is allowed for in these methods) used by major South African industrial companies tend to lead to under-investment.

2 INTRODUCTION

2.1 Inflation trends in South Africa[1]

In the decade from 1961 to 1970 the annual rate of inflation in South Africa never reached 5%. At this level the actual inflation rate and expectations of future inflation rates were probably not important factors in business investment decisions. The decade from 1971 to 1980 presents a completely different picture. By 1973 the inflation rate had reached 10% p.a. and it has remained a double-digit number since then. Since 1976 it has been on a rising trend with the figure for 1979 being 14% and that for 1980 being 15,8%.

Current inflation rates and anticipated future inflation rates are now an important factor in business investment decisions. This paper was motivated by the desire to establish how the inflation factor was incorporated into the investment decision by major South African industrial companies. It was decided to follow the approach used in a previous British study details of which follow.

2.2 The Carsberg and Hope Study in the UK

In 1976 the Institute of Chartered Accountants in England and Wales published a book by Bryan Carsberg and Tony Hope, both of the University of Manchester, entitled 'Business Investment Decisions Under Inflation'. The contents of this book can be divided into three parts. Firstly there is the motivation of the study (Chapter 1). Then follows a short review, analysis and summary of the theory underlying investment appraisel methods (Chapters 2 – 6).

Finally, there are the results of an investigation of the methods of investment appraisal used in practice and conclusions arising therefrom (Chapters 7 – 9).

In so far as the motivation of the study is concerned the authors state "Our specific purpose is to evaluate the hypothesis that the investment appraisal practices commonly used in British firms contribute to a tendency to under-investment" (p. 2).

To this end the book "summarises and illustrates modern views on an optimal method of appraisal under inflation, ignoring risk" (p. 2), "extends the analysis to take account of risk" (p. 3), "deals with the selection of the acceptance criterion – the cost of capital" (p. 3) and "considers some alternative methods of appraisal and evaluates their relative effects on the level of investment" (p. 3). Thereafter these conclusions regarding the theory are brought together in a form suitable for comparison with the empirical findings.

These conclusions regarding the theory can be summarised as follows:

1 "...the best method of appraisal involves the calculation of net present values and there are two equivalent methods of detailed calculation: the discounting of real cash flows at a real required rate of return or the discounting of money cash flows at a money required rate of return" (p. 38).

2 "A similar situation would prevail if the internal rate of return method were used. The rate for the project and the target rate should both be in money terms or both in real terms" (p. 38).

3 "...the fundamental properties of the internal rate of return method may lead to bias against the acceptance of projects requiring large investments, in the case of competition between alternative projects, regardless of the treatment of inflation" (p. 38).

The empirical evidence was obtained by means of a postal questionnaire survey. "The questionnaire was sent, in the Autumn of 1973, to a sample of 325 companies chosen randomly from The Times list of 1 000 leading UK companies for 1971-72" (p. 40). Analysis of this evidence in the light of the previously established conclusions regarding the theory lead the authors to conclude as follows, subject to the normal reservations necessary when evidence has been obtained in this way:

"Nevertheless, we believe that the evidence accumulated in our study provides strong support for the hypothesis that investment appraisal practices used by large British firms tend to lead to investment below the optimal level.

"A large number of firms use a money target rate of return in association with cash flows estimated in current prices (with the added disadvantage of

[1] Based on end December values for the seasonally adjusted Consumer Price Index taken from the Quarterly Bulletin of the South African Reserve Bank.

failure to predict the effects of differential inflation as it affects their resources). Several firms use the internal rate of return as a method of appraisal and others use a first year accounting rate of return. Both methods in comparison with the net present value method, are likely to lead to under-investment. Several firms also use the payback method which appears to reflect an unduly cautious approach to investment decisions. To offset these factors we have identified only the use of the average rate of accounting return method which may involve a relatively slight bias to over-investment." (p. 64).

3 GENERALISABILITY OF THE CARSBERG AND HOPE CONCLUSIONS TO SOUTH AFRICA

In 1978 the writer read the Carsberg and Hope monograph with interest and decided to find out whether the conclusions reached were applicable in South Africa. Professor Carsberg kindly supplied a copy of the questionnaire used in their study. With very minor changes in wording this questionnaire was used to obtain from South African business firms evidence with which to test the hypothesis *that the investment appraisal practices used by large South African industrial firms tend to lead to investment below the optimal level.*

4 METHOD

4.1 Subjects utilised and instruments used

The subjects utilised were the top 100 South African industrial companies given in the April 1977 edition of the Financial Mail's Top Companies report. The industrial companies utilised were ranked by asset size.

The instrument used to obtain evidence from the subjects was the questionnaire designed by Carsberg and Hope. The questionnaire included 16 questions and covered 5 typewritten A4 pages. The majority of the questions were of the multiple choice type, a few required brief comments or description and one required a ranking on a scale from 1 – 7.

4.2 Research procedure

In February 1978 the questionnaire together with a covering letter was sent to the chairman of the board of the subject companies. The covering letter briefly explained the purpose of the research making reference to the Carsberg and Hope study. It requested participation by the company and suggested that the chairman direct the questionnaire to the executive responsible for investment appraisal for completion. A guarantee of confidentiality of individual practices was given.

50 usable replies were received, a response rate of 50% compared with the Carsberg and Hope response rate of 32%. From now on the Carsberg and Hope results are given in brackets after the South African results. 19 companies returned the questionnaire uncompleted. Of these 9 said they did not have the necessary executive time, 7 thought the questionnaire inappropriate to their type of business (e.g. they were too diversified), and 3 were now subsidiaries of other companies.

The questionnaire included two questions designed to test the respondent teams' interests in this research. One asked whether the firm would agree to having a short meeting with the writer to discuss their investment appraisal methods in greater detail.

The responses were as follows:

	SA %	UK %
(a) I would positively welcome a meeting	14	11
(b) I would agree to a meeting	46	49
(c) I would not agree to a meeting	40	40
	100%	100%

Furthermore in response to another question 80% of respondents (82%) requested a summary of the research findings. An interim summary was provided to these firms in July 1978.

These responses showed a remarkable similarity to the UK experience and suggest that the respondent firms had a genuine interest in the topic of the research. The implication of this is that they would complete the questionnaire with due care.

5 RESULTS AND DISCUSSION

5.1 Choice of investment appraisal method

In this area the reader will notice a remarkable similarity between the 1978 South African responses and the 1973 UK responses. It was established that 78% of South African and 71% of UK firms used more than one method of appraisal. The popularity of the various methods was derived from the responses to question 1. These questions are summarised in *Table 5.1* overleaf.

Before proceeding to draw any conclusions from the information in Table 5.1 a brief description of its construction is in order. Respondents were requested to rank the importance of the various investment appraisal methods to their company on a scale from 1 – 7. In this case, one means, "used as the main criterion on virtually all projects", whereas seven means, "used occasionally as a subsidiary criterion". From this information an initial ranking of the methods, entitled "Points score ranking" was obtained as follows:

A "points score" was calculated for each method by multiplying the number of responses ranking a method one by one, the number of responses ranking a method two by two and then summing these products. A low points score would indicate a widely used method and vice versa.

From this number the first ranking of the popularity of the various investment appraisal methods is obtained.

The next column also reflects a popularity ranking in this case based not on a weighted average but only on the number of times this method was nominated as the primary (No. 1) method. The final column is also a popularity ranking, this time based on the number of respondents who did not use the method at all.

It is interesting to note that the ranking systems produce more or less the same rankings and that these give very similar indications on the level of popularity of the various investment appraisal methods in the two countries. Internal rate of return ranked as the most widely used method in SA, whilst it ranked second in the UK. Qualitative judgement ranked second in SA, but first in the UK. The

Table 5.1 – Popularity of investment appraisal methods

Method	Country	Points score ranking	Primary method ranking	Not used ranking
Payback period – without discounting	SA	3rd	Joint 3rd	3rd
	UK	3rd	3rd	3rd
Payback period – with discounting	SA	Joint 4th	Joint 4th	Joint 4th
	UK	Joint 4th	Joint 4th	Joint 5th
ARR. First year profit/ Initial Inv.	SA	Joint 4th	Joint 4th	Joint 5th
	UK	Joint 4th	Joint 4th	Joint 5th
ARR. Average profit/ Average Inv.	SA	Joint 4th	Joint 3rd	Joint 4th
	UK	Joint 4th	Joint 4th	Joint 5th
Net present value (NPV)	SA	Joint 4th	Joint 4th	Joint 5th
	UK	Joint 4th	Joint 4th	Joint 4th
Internal rate of return (DCF Yield)	SA	1st	1st	2nd
	UK	2nd	2nd	2nd
Net terminal value	SA	5th	5th	Joint 5th
	UK	5th	5th	Joint 5th
Qualitative judgement	SA	2nd	2nd	1st
	UK	1st	1st	1st

traditional payback method ranked 3rd in both countries. Also, in SA, the traditional ARR measure (average profit/average investment was quite widely used).

Despite the high ranking given to the qualitative judgement criteria by managers in both countries, it is clear that numerical calculations are still of fundamental importance in the investment decision. 72% of SA managers and 75% of UK managers responded this way. This conclusion is further strengthened by the finding that the average percentage of projects accepted was only 61% (75%), and only 12% (23%) of respondents indicated acceptance rates of 90% or more.[2]

Further observations in this area are that despite the wide (84% SA, and 85% UK) use of DCF methods these are supplementing rather than displacing traditional methods such as payback and ARR which are still widely used. Also of the two main DCF methods the internal rate of return method enjoys considerably wider usage than its theoretically more acceptable counterpart, the net present value method.

[2] These percentages would be based on the number of projects formally appraised by top management. It might be interesting to consider how many projects are eliminated by informal processes lower down the management structure.

5.2 Treatment of inflation in investment appraisal methods

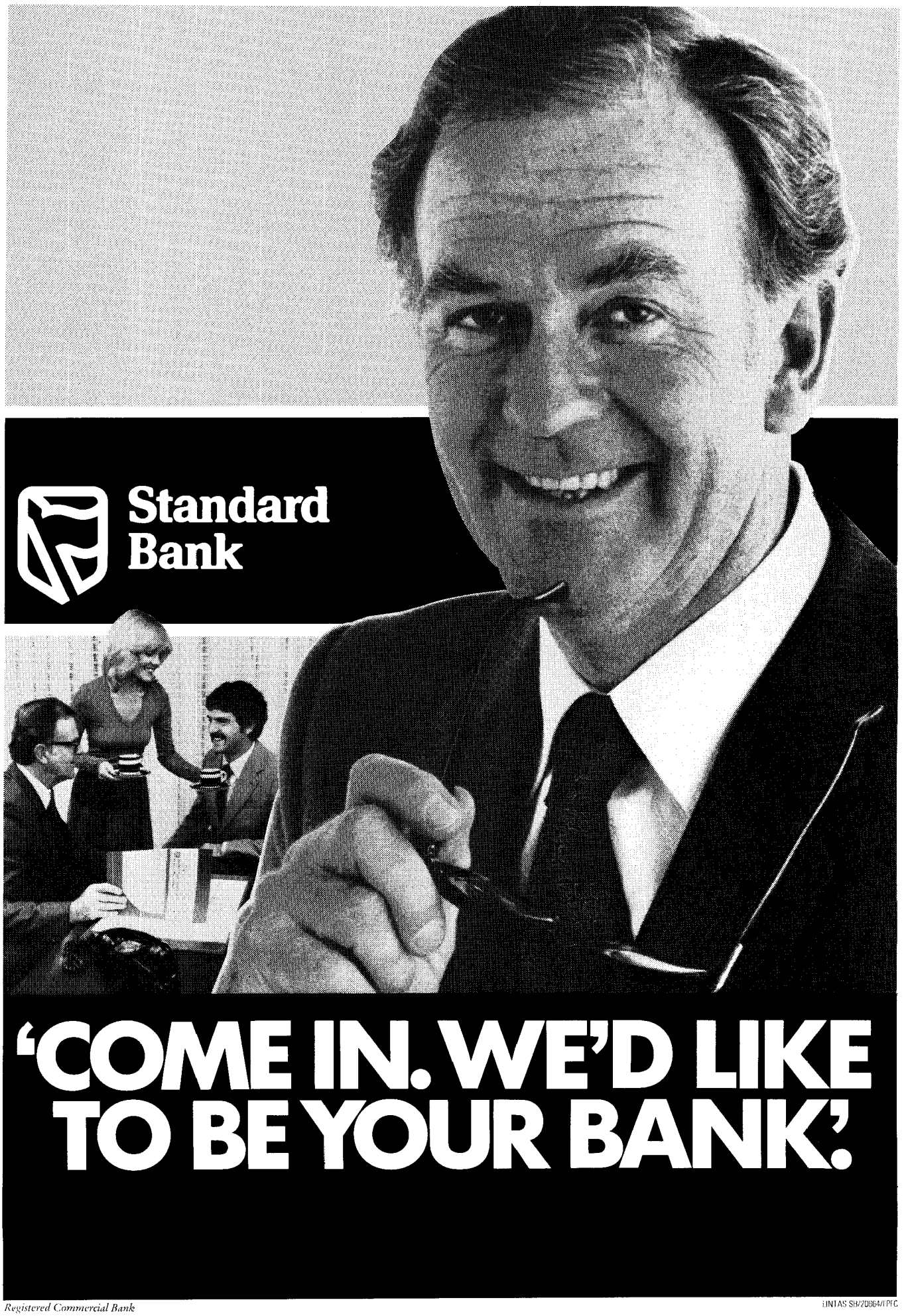
Estimation of the cost of capital

The arguments developed in the earlier part of the Carsberg and Hope book "indicate that optimal decisions involve calculating a net present value by discounting real cash flows at a real rate of interest or discounting money cash flows at a money rate of interest. Inflation should be treated consistently in the estimation of returns and in the setting of targets. Similar considerations apply to the use of the internal rate of return or the accounting rate of return." (p. 49).

In order to establish whether inflation was correctly treated in the methods used in practice firms were requested to supply, inter alia, details regarding the basic discount rate or the minimum (cut-off) rate of return used.

Table 5.2 – Information needed for aggregate WACC calculation

Year	Change in consumer price index (%)	Interest yield on company debentures (%)	Money cost net on 42% company tax (%)	Real cost net of 42% company tax (%)	SA industrials composite index dividend yield (money basis) (%)
1971	7,0	10,0	5,8	–1,1	6,4
1972	7,3	10,0	5,8	–1,4	6,0
1973	10,0	9,5	5,5	–4,1	4,8
1974	14,1	9,8	5,7	–7,3	7,2
1975	11,7	13,3	7,7	–3,6	9,9
1976	10,8	13,5	7,8	–2,7	10,1
1977	11,1	13,9	8,1	–2,7	11,5



Registered Commercial Bank

Before proceeding to summarise the responses it is useful to estimate, on an aggregate basis, the cost of capital of the South African industrial sector in 1978. This will be done using the traditional weighted average cost of capital (WACC) approach incorporating a constant dividend growth model for equity valuation. For this purpose the information in Table 5.2 has been collected.

The information contained in Table 5.2 was all obtained from publicly available sources and is more or less self-explanatory. From this information the component costs of the cost of capital can be obtained. 8% p.a. can be taken to be a fair estimate of the cost of debt. The cost of equity is estimated using the constant dividend growth model

$$(\text{i.e. } k_e = \frac{d_1}{P_0} + g)$$

where d_1 is the next dividend (in money terms) expected, P_0 is the current share price (in money terms) and g is the expected growth in dividends (in money terms) expressed as a percentage. Drawing from the table we estimate

$$\frac{d_1}{P_0}$$

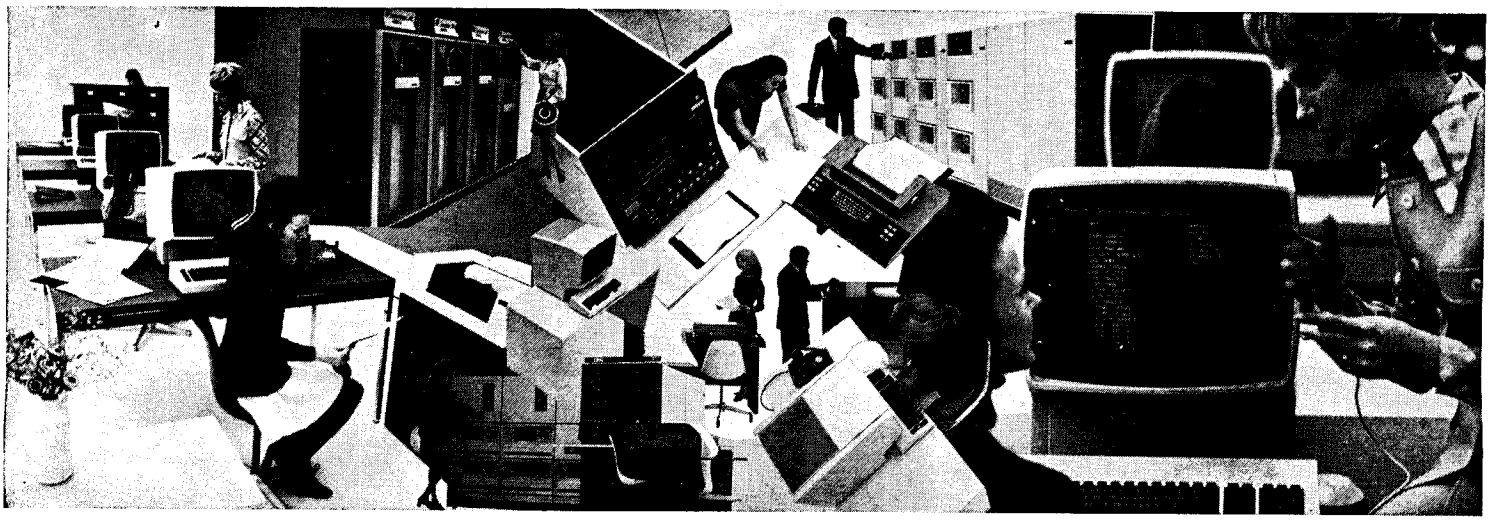
at 12% and g at 10% ($6{,}4 \times (1{,}10)^6 \simeq 11{,}5$) giving a value for k_e at 22%. Making the assumption that the typical South African industrial company is financed one-half by debt and one-half equity, we arrive at a weighted average cost of capital suitable for use as an investment decision discount rate of about 15% (WACC = ($\frac{1}{2}$ x 8%) + ($\frac{1}{2}$ x 22%) = 15% in money terms or about 3,5% in real terms after allowing for the effects of inflation. This value can now be used as a benchmark when considering the values actually used by the companies as a discount or cut-off rate.

The responses regarding the level of target rate of return or discount rate and the period of time which has elapsed since it was last altered is summarised in Table 5.3. Inspection of Table 5.3 indicates a range of values for required rate of return from 9% to 30% with a model value of 15%. Clearly then, respondents have submitted their figures in money terms rather than real terms and hence cash flows or accounting profits should also be specified in these terms for investment decision purposes.

Table 5.3 – Level of target rate of return or discount rate and time since last altered

Rate used (%)	Number of companies using rate	Rate chosen as a matter of policy	Time since rate last altered (in years) <1	1	2	3	4	5	6+	Not disclosed
9	3	–			2					1
10	1	1		1						
12	5	5			2	1	1			1
13	1			1						
14	1	1						1		
15	9	3	1	3		2			1	2
16	1	1								1
17	4	3		1	1	2				
18	1	1	1							
20	3	2		1	1					1
22	1	–	1							
23	1	1		1						
25	2	2		2						
30	1	1						1		
Not disclosed	16	8	2	4	1					9
	50	29	5	14	7	5	1	2	1	15
		SA companies as %	10	28	14	10	2	4	2	30
		UK companies as %	7	10	13	13	2	10	9	36

There seems to be a slight tendency for firms which have more recently changed their rates to have adopted higher rates. Also over half of the SA companies responding to this question indicate that they had altered their rates within the last year. This is much higher than the UK situation of 5 years prior. Taking the aggregate cost of capital figure previously calculated at 15% as a benchmark one could accept that rates up to 20% could possibly be justified as basic rates. However, rates above this are probably too high to be basic rates and would produce a bias towards under-investment. Five companies reported using rates above 20% and would therefore fall into this category.



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Also included in Table 5.3 is an indication that a majority of companies (about 60%) choose their discount rate as a matter of management policy rather than in some objective and readily explained way. Discount rates chosen in this subjective way varied through the whole range of rate values.

Another popular basis (about 20% of respondents) for establishing a rate was the interest rates on bank overdrafts. The remaining 20% of respondents used an objective method which could possibly be optimal in the circumstances.

The firms were also asked whether the normal rate was varied for individual projects. 70% of respondents said that this was the case. They explained that the adjustment was usually based on top management judgement taking into account the nature of the project (8 firms), the risk of the project (12 firms), and strategic factors (14 firms). Finally the firms advised that in about 80% of cases the rate was established by the managers alone, whilst in the remainder of cases the advice of consultants or writers in the field was sought. This is very similar to the UK results.

The treatment of inflation

The information obtained in this regard is summarised in Table 5.4 which forms the heart of this section of the analysis.

Table 5.4 – The treatment of inflation in investment appraisal

(1) Total respondents SA/UK	(2) Basic target Rate of return SA/UK	(3) Adjustment to basic rate SA/UK	(4) Adjustment to cash flow or profit estimates SA/UK		(5) Outcome number
100%/100%	9% TO 10% p.a. 8%/15%	NO.ADJ.(NA) 8%/10%	NA	8%/ 4%	1
			IGI	0%/ 2%	2*
			ISI	0%/ 4%	3**
		INCR.GEN.INFL(IGI) 0%/4%	NA	0%/ 2%	4
			IGI	0%/ 1%	5
			ISI	0%/ 1%	6
		INCR.SPEC.INFL(ISI) 0%/1%	NA	0%/ 0%	7
			IGI	0%/ 0%	8
			ISI	0%/ 1%	9
	11% TO 14% p.a. 14%/18%	NO.ADJ.(NA) 12%/15%	NA	10%/13%	10
			IGI	0%/ 1%	11*
			ISI	2%/ 1%	12**
		INCR.GEN.INFL(IGI) 0%/2%	NA	0%/ 1%	13
			IGI	0%/ 1%	14
			ISI	0%/ 0%	15
		INCR.SPEC.INFL(ISI) 2%/0%	NA	2%/ 0%	16
			IGI	0%/ 0%	17
			ISI	0%/ 0%	18
	15% TO 30% p.a. 46%/17%	NO.ADJ.(NA) 34%/14%	NA	24%/13%	19
			IGI	4%/ 0%	20*
			ISI	6%/ 1%	21**
		INCR.GEN.INFL(IGI) 2%/3%	NA	0%/ 1%	22
			IGI	2%/ 1%	23
			ISI	0%/ 1%	24
		INCR.SPEC.INFL(ISI) 10%/0%	NA	2%/ 0%	25
			IGI	0%/ 0%	26
			ISI	8%/ 0%	27
	NOT DISCLOSED 32%/50%	NO.ADJ.(NA) 18%/42%	NA	16%/32%	28
			IGI	2%/ 2%	29*
			ISI	0%/ 8%	30**
		INCR.GEN.INFL(IGI) 8%/0%	NA	0%/ 0%	31
			IGI	6%/ 0%	32
			ISI	2%/ 0%	33
		INCR.SPEC.INFL(ISI) 6%/8%	NA	2%/ 4%	34
			IGI	0%/ 2%	35
			ISI	4%/ 2%	36

In the first column is shown the total respondents from each study in percentage terms. In the second column, the totals are divided into four groups according to the rate of return disclosed. These four groups are then further sub-divided into three groups each (in column three) according to whether or not they adjust the rate-of-return (per column two). It is suggested that there should be no adjustment to the basic rate of return to allow for inflation.

FINANCIAL STATEMENT

Turnover	R3 459 473 000
Group profit before taxation	R516 408 000
Group profit after taxation	R343 470 000
Group profit after taxation attributable to ordinary shareholders in Barlow Rand Limited	R221 855 000
Earnings per share before non-trading items	179,6 cents
Dividends per ordinary share	58,0 cents
Total assets	R2 928 314 000
Number of shareholders	24 644
Number of employees – subsidiaries	114 292
– managed associates	77 403

Salient features taken from 1980 Annual Financial Statements.

SOCIAL STATEMENT

A company's first duty is to its shareholders.
But a company our size has another responsibility.
To the 193 000 people who work for us.
That's why we've made a commitment –

* To work for changes in any laws and attitudes that result in discrimination against employees in the work situation.
* To appoint and promote people according to ability and achievement. And not according to colour.
* To remunerate employees on a non-discriminatory basis. Experience, performance, and the nature of the job being the determining factors.
* To continue our comprehensive training and development programmes. Not only to ensure our progress. But to equip our employees for a better future with better opportunities.
* To provide, where necessary, financial assistance for the education of our employees' dependants.
* To continue to finance the development of education and training institutions through our C.S. Barlow Foundation.
* To improve the quality of life of our people.

All these commitments – and others – are embodied in our own Code of Employment Practice which we adopted in 1978. It's a living document. And we regularly monitor the progress our companies are making in its implementation.

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BARLOW RAND LIMITED

This is because the majority of the discount rates disclosed are within an acceptable range of the benchmark money cost of capital figure of 15%. This figure has been derived from numbers obtained from the capital markets and thus has impounded in it the capital market's inflation expectations. Note that 72% (8% + 12% + 34% + 18%) of the SA companies make this decision correctly compared with 81% (10% + 15% + 14% + 42%) of the UK companies. The balance are rates which are probably too high and thus create a bias toward under-investment.

In column four, the existing twelve groups from column three are further subdivided according to whether or not the respondent company adjusts its cash flow or profit estimates for inflation, and if it does, whether this adjustment is in respect of the general or specific level of expected inflation. The theory supports the view that an adjustment should be made in respect of specific expected inflation. Ideally then, the cash flow or profit estimates used in the investment appraisal method will be the actual money amounts that will arise when the investment is carried out (other things being equal!). These arguments would suggest that outcomes 3, 12, 21 and 30 (marked xx on Table 5.4) represent optimal adjustments to the investment appraisal method in respect of expected inflation. Inspection of the table shows that only 8% (0% + 2% + 6% + 0%) of SA companies did this and in the UK only 14% (4% + 1% + 1% + 8%). This would suggest a strong bias toward under-investment in both countries.

It could be argued that an adjustment of cash flows or profits in respect of the general level of expected inflation (outcomes 2, 11, 20 and 29 marked x on Table 5.4) is a good enough approximation in practice. In that case only a further 6% (0% + 0% + 4% + 2%) of SA companies can be said to adjust for inflation in a manner which approaches the optimal (UK 5% (2% + 1% + 0% + 2%)). In other words 86% of large SA industrial companies incorporate inflation into their investment appraisal methods in a way which would give a bias towards under-investment. This comparative figure for the UK in 1975 is 81%.

6 IMPLICATIONS AND CONCLUSIONS

The main conclusion from this study is that it gives strong support to the hypothesis (stated on page 9 that the investment appraisal practices used by large South African industrial firms will tend to lead to investment below the optimal level.

The main factors giving rise to this are:

– The majority of firms use a money discount rate of return in conjunction with cash flows or profits estimated in current terms. This would probably lead to under-investment in inflationary times regardless of the investment appraisal method used.

Furthermore

– Many firms use the internal rate of return method as their primary method of investment appraisal. This method is likely to lead to under-investment by comparison with the theoretically preferable net present value method.

– Many firms use the payback method which would suggest an overly conservative approach to investment decisions.

In a study of this nature it is never possible to be sure whether one has asked the right questions or interpreted the answers correctly. If, on balance, this has been achieved in this study, then the results are of some importance to the business community. One form of action that would seem to be desirable would be to check the results of these studies, preferably by means of further in-depth studies. The writer also supports one of Carsberg and Hope's concluding recommendations which suggested that "A requirement that companies should publish a statement of the methods of investment appraisal that they use, and the level of their target returns would provide useful information to shareholders, and perhaps lead over time to a greater uniformity in use of best practice". (p. 65).

ACKNOWLEDGEMENTS

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	Year ended 31 March 1981	Year ended 31 March 1980	% Change
Net income after provision for taxation and transfers to contingency reserve	R2 500 000	R1 500 000	+66,7
Which has been dealt with as follows:			
– Transfer to general reserve	R1 495 000	R895 000	+67,6
– Dividends declared	R1 000 000	R600 000	+66,7
Total assets	R 255,0m	R 191,9m	+32,9
Shareholders' funds	R 16,5m	R 10,9m	+65,0
Net income as a % of average shareholders' funds	17,3%	16,6%	

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By Stanley Diller

Analyzing the yield curve: a new approach

This article originally appeared in the Financial Analysts Journal, March–April 1981. It is reproduced here with the kind permission of both the FAJ and the author.

When investors choose between a five and 10-year bond, they compare expected rates of return over the near future. Whether the five-year return on the 10-year bond is to equal the locked-in return on the five-year bond depends, however, on the yield on the 10-year bond at the end of five years. The initial yield on the 10-year bond may impound an expectation of a very different yield five years hence.

The yield on a long bond over the near future is the sum of its beginning yield to maturity and the annualized percentage price change resulting from the change in yield. The yield change that equates its total return with the locked-in return on an instrument maturing over that period the author terms the "breakeven yield change."

Sir John Hicks termed the fraction of expected change in short-term rates that is reflected in the expected change in short-term rates one period hence the "coefficient of expectation." The expected change in long interest rates will equal the average of expected changes in future short rates, calculated by applying the coefficient of expectation to the expected change in current short rates. The author calculates breakeven yield changes from observations of actual yield for various terms to maturity and then solves for the value of Hicks' coefficient that gives the best fit to the calculated breakeven yield change.

Actual data demonstrate that the pattern of breakeven yield changes is well behaved and smooth. On the other hand, when the best fitting patterns are translated back into the corresponding yield curves, they have the full variety of complex curvatures observed in reality.

As the core of fixed income analysis, the yield curve has gotten more attention than any other area of bond research. But it often seems that technology has outrun theory, and that complex algorithms have displaced insightful lines of inquiry. This article attempts to redress this imbalance by returning to the principles, now well accepted, nominally underlying much of the current work – namely, that the yield curve reflects people's *outlook,* rather than the relative supplies of bonds with different terms to maturity.

The question is, the outlook for what? The model we propose assumes that people choose between a five and 10-year bond, not by forecasting the five-year interest rate five years in the future, as academic studies have suggested, but by comparing the rates of return they expect on these bonds in the near future.This approach is more consistent with the main thrust of fixed income analysis, which has tended to concentrate on near-term performance. Furthermore, with turnover costs added, our model explains why yield curves are not the same across all sectors – why, for example, tax-exempt curves slope upward even when the Treasury curve is sharply inverted. Above all, it leads to a way of fitting smooth and accurate curves to yields on actual bonds.

Stanley Diller is Vice President of the Financial Strategies Group, Goldman, Sachs & Co.

Breakeven yield changes

To the extent that the change in yield over a short holding period is surprise, a long bond obviously has an element of investment risk over that period. But not all yield change over the period is necessarily surprise; the price – hence the yield – of the long bond at the beginning of the period may impound an expectation of a very different yield at the end. If investors price bonds so that their near-term performance is comparable, then the return on a long bond over the life of a bill should equal the risk-free, or locked-in, rate of return on the bill. Unlike the return on the bill, the return on the long bond over that shorter period will depend in part on the yield on the bond at the end of the period.

One expression for the rate of return (ROR) for a period spanning six months or less is given by:

$$ROR_N = Y_N + (\Delta P)(12/H), \quad (1)$$

where Y_N is the starting yield to maturity, ΔP the percentage change in price *associated with a yield change,* N the term of the bond and H the holding period in months. This equation says that the ROR on a bond within a single compounding period is equal to the sum of its starting yield to maturity and its *annualized* percentage price change *resulting from a yield change.* (The last phrase excludes aging toward par.)

The price change may be represented (and is generally thought of) as the product of a yield change (ΔY_N) and a factor (D_N) that is the bond's duration divided by $1+Y_N/2$ (hereafter called Duration):

$$\Delta P_N = D_N \Delta Y_N. \quad (2)$$

Substituting Equation (2) into Equation (1) gives:

$$ROR_N = Y_N + (D_N \Delta Y_N)(12/H). \quad (3)$$

A reasonable condition for buying a bond at risk is that its expected ROR at least equal the yield (Y_B) one can lock in with a security maturing at the horizon – say, a bill. Substituting Y_B for ROR in Equation (3), we have:

$$Y_B = Y_N + (D_N \Delta Y_N)(12/H). \quad (4)$$

The value of ΔY_N that equates the bond's return with the locked-in rate over the investment horizon is called the breakeven yield change (BYC_N):

$$Y_B = Y_N + (D_N BYC_N)(12/H). \quad (4A)$$

Rearranging terms, we obtain the formula for the breakeven yield change implied by the current bond and bill yields:

$$BYC_N = \frac{(Y_N - Y_B)(H/12)}{D_N}. \quad (5)$$

The right-hand expression is the de-annualized spread between the bond and bill rates divided by the bond's Duration.

Table I lists the BYCs on a given day for most Treasury bonds and several horizons. The top of each right-hand

column gives the horizon for the BYCs below, as well as the bill yield against which the bonds break even. By definition, the BYC for each bond brings about an ROR equal to the bill yield.

Why BYCs decay smoothly

As Table I shows, the absolute values (i.e., ignoring minus signs) of BYCs fall as terms rise. This is consistent with Sir John Hicks' theory of short-term changes in long-term rates.[1] Hicks argued that the market expects a change in short-term rates (perhaps brought about by monetary policy) to continue, at least in part. He defined the *coefficient of expectation* as the fraction of the original change that persists. The larger this coefficient, the greater the future changes in relation to the current change.

Future changes in short rates affect the current long rate, which (as Hicks defines it) is the average of the expected short rates (with a little added to compensate for greater risk). The expected change in the long rate, therefore, is the average of the expected changes in the short rates. In other words, people extrapolate changes in current short rates to changes in future short rates, which then change the current long rate.

In Hicks' model, expectations of changes in short rates fade as the rates become more remote. In the following calculations, each expected yield change is K times the one before it (K – Hicks' "coefficient of expectation" – being less than one). Thus the second change is K times the first, the third K times the second, or K^2 times the first, and the Nth change K^{N-1} times the first. Calling the first of this series $\Delta Y_{1,1}$, each successive change is $K\Delta Y_{1,1}$, $K^2\Delta Y_{1,1}$, ... $K^4\Delta Y_{1,1}$. More generally, the expected change in the one-period rate N periods in the future ($\Delta Y_{1,N}$) is:

$$\Delta Y_{1,N} \equiv K^{N-1}\Delta Y_{1,1}. \qquad (6)$$

These successive changes in expected future short rates translate into short-term changes in longer rates.

In theory, the one-period change in the N-term rate corresponds to the average of the consecutive forecast changes of the first N one-period rates. For example, the one-period change in the five-period rate is the average of

Table I
Breakeven yield changes, Bonds vs. Bills (basic points)

Report date: 11/6/80 — **Delivery date: 11/10/80**

			Bill maturities				
	Bond equivalent yields→		1-month 11.75	3-month 14.15	6-month 14.49	1-year 14.13	2-year 13.59
Coupon	Maturity	Yield					
7.375	05/15/81	14.150	27.0	–35.0	–526.5		
7.500	05/15/81	14.144	26.9	–35.5	–538.2		
9.750	05/31/81	14.324	30.5	–24.8	–141.6		
6.750	06/30/81	14.334	25.7	–17.4	–71.0		
9.125	06/30/81	14.366	26.3	–16.0	–60.8		
9.375	07/31/81	14.300	22.2	–12.8	–65.7		
7.625	08/15/81	14.459	22.2	–3.5	–32.9		
8.375	08/15/81	14.491	22.6	–2.0	–27.6		
9.625	08/31/81	14.335	22.3	2.1	–41.3		
10.125	09/30/81	14.112	17.4	–9.9	–58.2		
6.750	09/30/81	14.319	19.0	–1.9	–31.6		
12.625	10/31/81	13.961	15.3	–11.2	–48.7		
7.750	11/15/81	13.779	12.9	–16.7	–58.7	–1 215.4	
7.000	11/15/81	13.779	12.9	–16.7	–58.5	–1 216.8	
12.125	11/30/81	13.979	13.7	–13.4	–44.3	–231.5	
11.375	12/31/81	13.972	12.0	–14.2	–38.0	–124.8	
7.250	12/31/81	13.740	10.2	–19.8	–55.4	–273.0	
11.500	01/31/82	13.980	11.1	–14.2	–36.3	–82.9	
6.125	02/15/82	13.535	7.8	–24.0	–63.8	–229.5	
13.875	02/28/82	13.774	10.6	–12.8	–46.2	–132.4	
15.000	03/31/82	13.715	9.2	–14.9	–47.1	–119.8	
7.875	03/31/82	13.775	9.2	–13.0	–41.2	–100.2	
11.375	04/30/82	13.686	8.7	–12.6	–38.5	–97.4	
8.000	05/15/82	13.649	8.0	–13.0	–37.3	–92.6	
7.000	05/15/82	13.437	6.9	–17.0	–47.7	–133.7	
9.250	05/15/82	13.675	8.2	–12.5	–36.2	–87.8	
9.375	05/31/82	13.686	7.8	–14.0	–38.2	–80.0	
8.625	06/30/82	13.694	7.0	–14.1	–34.4	–68.5	
8.250	06/30/82	13.669	6.8	–14.5	–35.4	–72.3	
8.875	07/31/82	13.718	6.7	–14.0	–33.0	–57.8	
9.000	08/15/82	13.701	6.5	–14.1	–33.7	–57.7	
8.125	08/15/82	13.560	5.8	–16.3	–39.2	–76.4	
11.125	08/31/82	13.529	6.6	–12.9	–38.9	–79.3	

8.375	09/30/82	13.577	6.1	−12.4	−34.6	−65.6	
11.875	09/30/82	13.574	6.2	−12.8	−36.0	−67.9	
12.125	10/31/82	13.588	6.3	−10.8	−30.0	−60.2	
7.875	11/15/82	13.412	5.2	−12.9	−33.8	−73.0	−925.6
7.125	11/15/82	13.390	5.1	−13.2	−34.4	−74.9	−1 040.8
9.375	12/31/82	13.150	3.5	−18.2	−42.4	−90.9	−572.8
8.000	02/15/83	13.275	3.7	−16.2	−37.3	−71.5	−249.3
9.250	03/31/83	13.156	3.6	−15.0	−38.4	−76.5	−238.1
11.625	05/15/83	13.061	3.3	−14.9	−36.7	−77.8	−218.7
7.875	05/15/83	13.122	3.4	−13.6	−33.8	−70.8	−189.8
8.875	06/30/83	13.031	2.5	−16.0	−36.2	−72.7	−185.8
11.875	08/15/83	13.156	2.8	−15.2	−34.1	−62.5	−125.7
9.250	08/15/83	13.257	3.0	−13.7	−30.7	−54.5	−95.4
9.750	09/30/83	13.214	3.2	−12.0	−30.0	−54.7	−94.9
7.000	11/15/83	12.964	2.4	−12.8	−30.8	−62.5	−133.5
9.875	11/15/83	13.393	3.6	−9.0	−22.2	−40.6	−42.9
10.500	12/31/83	13.182	2.6	−12.5	−27.4	−50.2	−80.6
7.250	02/15/84	13.349	2.8	−10.6	−23.2	−37.9	−43.2
14.250	03/31/84	13.480	3.6	−8.4	−21.5	−33.5	−18.9
9.250	05/15/84	13.217	2.8	−9.3	−22.2	−41.2	−56.2
13.250	05/15/84	13.454	3.5	−7.6	−18.6	−31.8	−20.1
8.875	06/30/84	13.213	2.3	−10.4	−22.6	−39.6	−52.7
7.250	08/15/84	13.300	2.4	−9.7	−21.0	−33.8	−38.0
12.125	09/30/84	13.382	2.9	−8.1	−20.1	−32.0	−27.2
8.000	02/15/85	12.952	1.5	−11.2	−24.2	−42.1	−66.3
10.375	05/15/85	13.076	2.0	−8.6	−20.0	−36.5	−50.1
14.375	05/15/85	13.317	2.6	−7.3	−17.3	−29.7	−27.4
8.250	08/15/85	12.874	1.2	−10.8	−23.0	−40.0	−62.7
9.625	08/15/85	13.156	1.8	−9.2	−19.6	−31.8	−39.0
11.750	11/15/85	13.196	2.1	−7.3	−17.0	−29.7	−33.5
7.875	05/15/86	13.040	1.6	−7.2	−16.6	−29.6	−38.9
8.000	08/15/86	12.926	1.1	−8.9	−18.9	−31.7	−44.7
9.000	02/15/87	12.897	1.1	−8.7	−18.4	−30.7	−43.1
12.000	05/15/87	12.971	1.4	−7.2	−16.5	−29.4	−38.7
7.625	11/15/87	12.965	1.2	−6.3	−14.4	−25.4	−33.1
8.250	05/15/88	12.879	1.1	−6.5	−14.7	−26.2	−35.6
8.750	11/15/88	12.963	1.2	−6.0	−13.5	−23.5	−29.9
9.250	05/15/89	12.906	1.1	−6.0	−13.6	−23.9	−31.2
10.750	11/15/89	13.013	1.2	−5.6	−12.6	−21.7	−26.0
8.250	05/15/90	12.508	0.6	−7.1	−15.6	−28.5	−43.6
10.750	08/15/90	12.967	0.9	−6.5	−13.6	−21.7	−26.6
1.000	11/15/90	13.121	1.3	−5.1	−11.4	−19.2	−20.3
7.250	08/15/92	12.967	0.8	−5.5	−11.3	−17.8	−21.3
7.875	02/15/93	13.032	0.8	−5.3	−10.8	−16.8	−19.0
8.625	08/15/93	13.011	0.8	−5.3	−11.0	−17.2	−19.7
8.625	11/15/93	13.013	1.0	−4.5	−10.0	−16.9	−19.3
9.000	02/15/94	12.997	0.8	−5.4	−11.1	−17.3	−20.0
8.750	08/15/94	13.010	0.8	−5.2	−10.8	−16.7	−19.1
10.125	11/15/94	13.053	1.0	−4.4	−9.8	−16.3	−18.0
10.500	02/15/95	13.030	0.8	−5.3	−10.9	−16.9	−18.9
10.375	05/15/95	13.042	1.0	−4.4	−9.8	−16.4	−18.1
12.625	05/15/95	13.151	1.1	−4.2	−9.4	−15.4	−15.2
11.500	11/15/95	13.075	1.0	−4.3	−9.6	−16.0	−17.2
8.500	05/15/99	12.902	0.8	−4.3	−9.6	−16.4	−19.9
7.875	02/15/00	12.879	0.6	−5.1	−10.4	−16.4	−20.1
8.375	08/15/00	12.872	0.6	−5.1	−10.5	−16.6	−20.5
8.000	08/15/01	12.889	0.6	−5.0	−10.2	−16.1	−19.6
8.250	05/15/05	12.732	0.6	−4.6	−10.2	−17.6	−23.2
7.625	02/15/07	12.517	0.3	−5.8	−11.9	−19.9	−28.2
7.875	11/15/07	12.480	0.4	−5.2	−11.4	−20.2	−29.1
8.375	08/15/08	12.589	0.3	−5.7	−11.7	−19.3	−26.7
8.750	11/15/08	12.634	0.5	−4.9	−10.7	−18.7	−25.6
9.125	05/15/09	12.667	0.5	−4.8	−10.6	−18.4	−24.9
10.375	11/15/09	12.751	0.6	−4.6	−10.2	−17.7	−23.0
11.750	02/15/10	12.867	0.6	−5.1	−10.5	−16.6	−20.4
10.000	05/15/10	12.687	0.6	−4.8	−10.5	−18.3	−24.6

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the expected changes in the one-period rate between the first and second periods, the second and third, and so on. The average of these changes is:

$$\left(\frac{1+K+K^2+K^3+K^4}{5}\right)\Delta Y_{1,1},$$

or

$$W_5 \Delta Y_{1,1},$$

where W_5 is the ratio of the one-period changes in the five and one-period rates. Hence the one-period change in the five-period rate is given by:

$$\Delta Y_{5,1} = W_5 \Delta Y_{1,1}.$$

More generally, for the N-term spot rate:

$$\Delta Y_{N,1} = W_N \Delta Y_{1,1}. \quad (7)$$

where

$$W_N = \frac{1+K+K^2+\ldots+K^{N-1}}{N} \quad (8)$$

Chart A Fitted Breakeven Values (April 28, 1980)

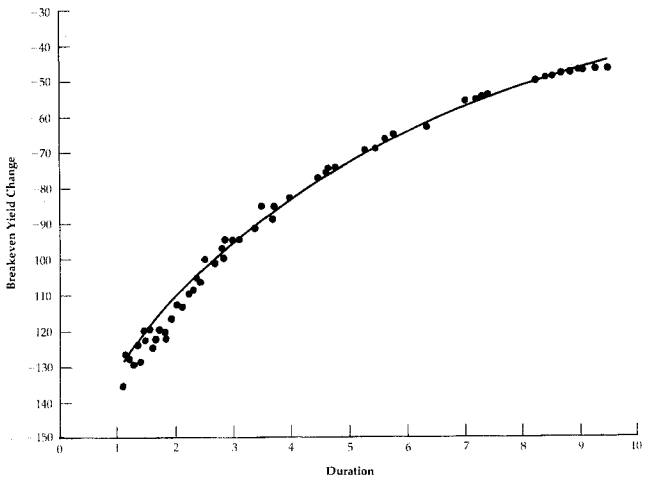
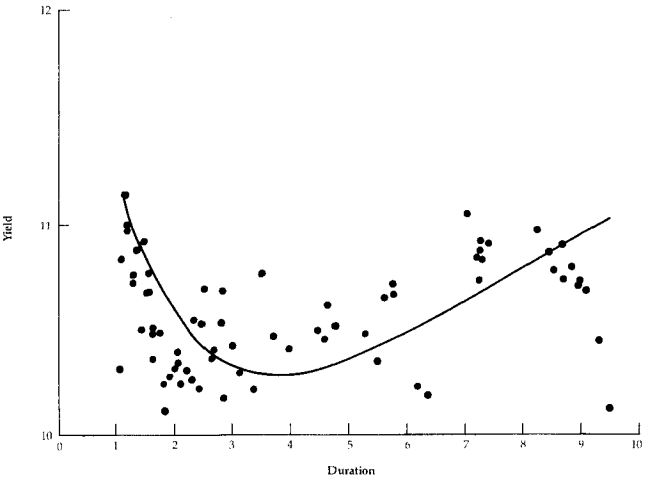


Chart B Yield Curve Implied by Breakeven Fit (April 28, 1980)



Applying the formula for the sum of a geometric progression gives us:

$$W_N = \frac{1-K^N}{N(1-K)}. \quad (8A)$$

As N gets larger, W_N gets smaller, and $\Delta Y_{N,1}$ falls relative to $\Delta Y_{1,1}$. (Hereafter, the second subscript, signifying the period of change, will be dropped, since only one-period changes of the long rates will be considered.)

Field yield curves

We have assumed long bonds are priced so that investors are indifferent between holding bills and holding long bonds over the remaining life of the bills – i.e., so that investors break even at the expected yield change on the bonds. We'll assume further that, across the range of bond maturities, yield changes with this breakeven property obey Hicks' model. Then we can substitute BYC_N for ΔY_N in Equation (7), obtaining:

$$BYC_N = W_N BYC_1, \quad (9)$$

or, substituting from Equation (8A):

$$BYC_N = \left[\frac{1-K^N}{N(1-K)}\right] BYC_1, \quad (10)$$

where the bracketed expression equals W_N, the ratio between the Nth and first BYC, and K is Hicks' coefficient of expectation.[2]

Equation (10) is used to smooth into an exponential fit the BYCs calculated by Equation (5) from actual yields. This is done by finding the values of three parameters – K, BYC_1 (the BYC on the shortest bond) and Y_B (the locked-in rate) – that minimize the squared deviations of the actual BYC_N from their smoothed counterparts.

Chart A shows the fitted BYCs for a particular day. Since the pattern of the raw data is very smooth (and remarkably similar under all kinds of market conditions), no special art or complexity is needed to fit it. Clearly, fitting these data is much easier than fitting the yields themselves, although the goal is a curve through the yields, rather than through the BYCs.

A curve through actual yields

To transform the BYC curve into a yield curve, we reverse the terms Y_B and Y_N in Equation (4A) to give:

$$Y_N = Y_B - (D_N BYC_N)(12/H), \quad (11)$$

which expresses the bond yield as the difference between the risk-free yield and the annualized product of Duration and the BYC. (When the bond yield, Y_N, is greater than the bill yield, Y_B, the sign on the BYC is positive; when it is less, the sign is negative. Duration is always positive.) This equation is a tautology that gains substance when the fitted BYCs replace the actual ones, so that:

$$\hat{Y}_N = Y_B - (D_N \hat{BYC}_N)(12/H), \quad (12)$$

where the supercripts signify fitted, rather than actual, data.

Chart B shows the yield curve that results from the fitted BYC curve in Chart A. Even more remarkable than the quality of fit is the curve's ability to change direction despite its derivation from a fitted BYC curve forced to go in one direction. Many people think a theoretically sound yield curve must go in one direction, on the grounds that it embodies remote forecasts that are unlikely to change direction. In our model, which also assumes unidirectional forecasting, the fitted yield curve *can* change direction for

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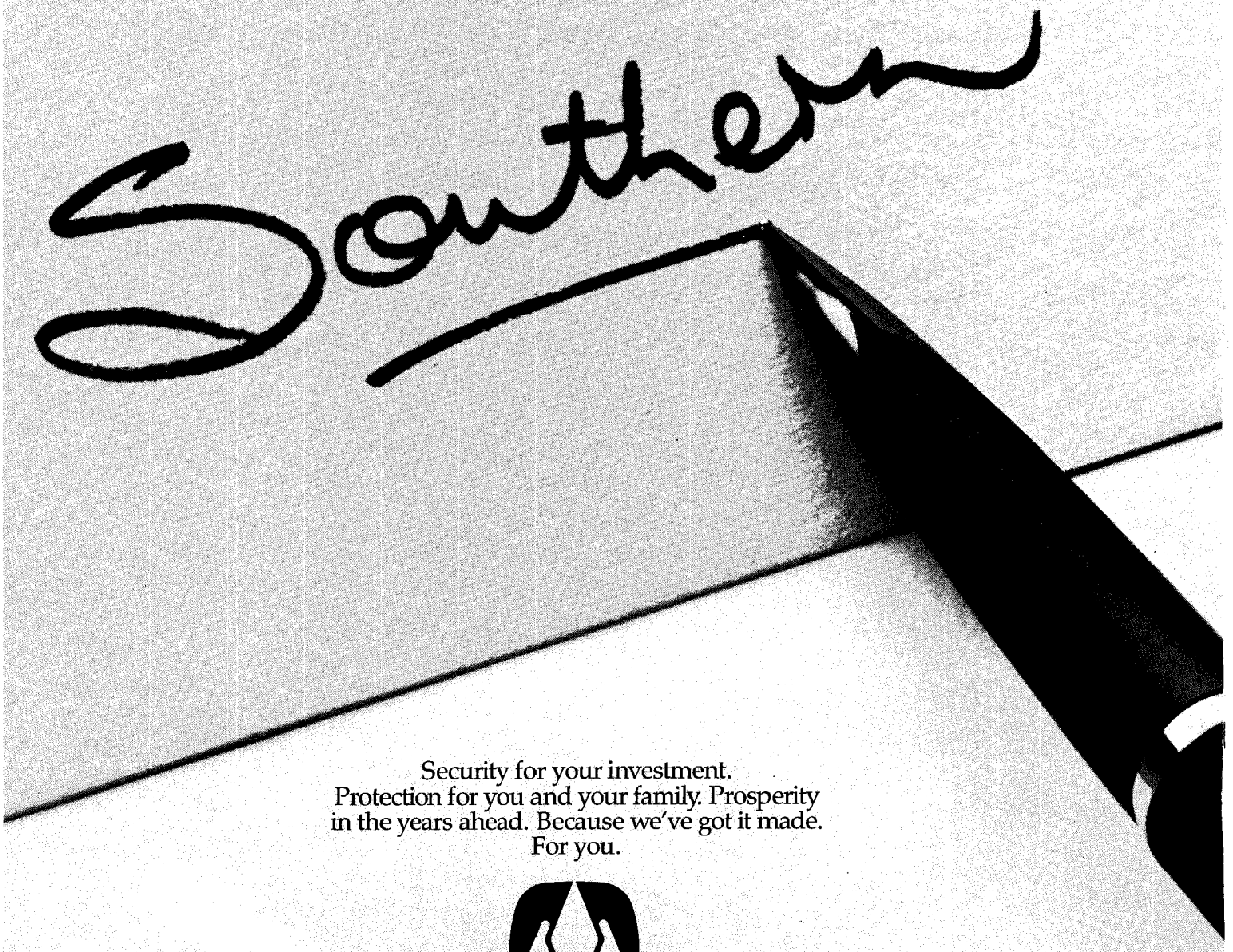

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reasons that are made clearer when Equation (8A) is substituted into Equation (12) to give:

$$\hat{Y}_N = Y_B - [(D_N W_N) B\hat{Y}C_1](12/H). \quad (13)$$

As maturity increases, D rises faster than W falls, making DW rise as N gets larger. In that case, the yield curve rises when BYC is positive and falls when it is negative. But in some cases, D rises first more quickly, then more slowly, than W falls, making DW first rise, then fall. The result is a "humped" or a "U"-shaped curve for positive and negative BYCs, respectively.

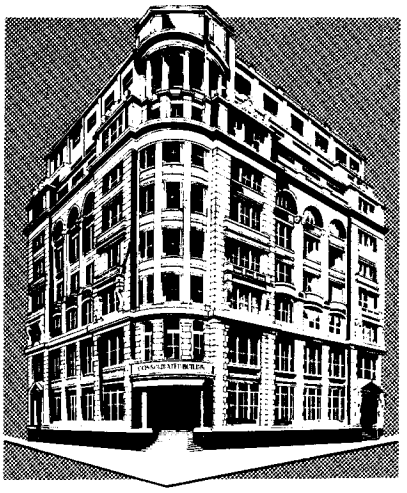
Table II shows the Ws for different values of Duration and

Table II
Values of W for various values of K and duration*

	Weights									
	Duration									
K	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.	10.0
0.9996	0.995	0.985	0.975	0.965	0.955	0.945	0.935	0.926	0.917	0.907
0.9992	0.990	0.970	0.950	0.931	0.912	0.894	0.876	0.859	0.842	0.826
0.9988	0.985	0.955	0.926	0.899	0.872	0.846	0.822	0.799	0.776	0.754
0.9984	0.980	0.941	0.904	0.868	0.834	0.802	0.772	0.744	0.717	0.691
0.9980	0.975	0.927	0.881	0.839	0.798	0.761	0.727	0.494	0.664	0.635
0.9976	0.971	0.913	0.860	0.811	0.765	0.723	0.685	0.649	0.616	0.585
0.9972	0.966	0.899	0.839	0.784	0.733	0.688	0.646	0.608	0.573	0.541
0.9968	0.961	0.886	0.819	0.759	0.704	0.655	0.611	0.571	0.535	0.502
0.9964	0.956	0.873	0.800	0.735	0.676	0.624	0.578	0.537	0.500	0.467
0.9960	0.952	0.860	0.781	0.711	0.649	0.595	0.548	0.506	0.469	0.436
0.9956	0.947	0.848	0.763	0.689	0.624	0.568	0.520	0.478	0.440	0.407
0.9952	0.942	0.836	0.745	0.668	0.600	0.543	0.494	0.452	0.414	0.382
0.9948	0.938	0.824	0.728	0.647	0.578	0.520	0.470	0.428	0.391	0.359
0.9944	0.933	0.812	0.712	0.628	0.557	0.498	0.448	0.406	0.370	0.338
0.9940	0.928	0.800	0.696	0.609	0.537	0.477	0.428	0.386	0.350	0.320
0.9936	0.924	0.789	0.680	0.592	0.518	0.458	0.409	0.367	0.332	0.302
0.9932	0.919	0.778	0.665	0.575	0.500	0.440	0.391	0.350	0.316	0.287
0.9928	0.915	0.767	0.651	0.558	0.483	0.423	0.374	0.334	0.301	0.273
0.9924	0.911	0.757	0.637	0.543	0.467	0.407	0.359	0.319	0.287	0.260
0.9920	0.906	0.746	0.623	0.528	0.451	0.392	0.344	0.306	0.274	0.248
0.9916	0.902	0.736	0.610	0.513	0.437	0.378	0.331	0.293	0.262	0.237
0.9912	0.897	0.726	0.597	0.499	0.423	0.365	0.318	0.281	0.251	0.227
0.9908	0.893	0.716	0.585	0.486	0.410	0.352	0.307	0.270	0.241	0.217
0.9904	0.889	0.706	0.573	0.473	0.397	0.340	0.296	0.260	0.232	0.209
0.9900	0.884	0.697	0.561	0.461	0.385	0.329	0.285	0.251	0.223	0.201
0]9896	0.880	0.687	0.550	0.449	0.374	0.318	0.275	0.242	0.215	0.193
0.9892	0.876	0.678	0.539	0.438	0.363	0.308	0.266	0.233	0.207	0.186
0.9888	0.872	0.669	0.528	0.427	0.353	0.299	0.258	0.226	0.200	0.180
0.9884	0.868	0.660	0.518	0.417	0.343	0.290	0.249	0.218	0.193	0.174
0.9880	0.863	0.652	0.508	0.407	0.334	0.281	0.242	0.211	0.187	0.168
0.9876	0.859	0.643	0.498	0.397	0.325	0.273	0.234	0.205	0.181	0.163
0.9872	0.855	0.635	0.488	0.388	0.316	0.265	0.227	0.198	0.176	0.158
0.9868	0.851	0.627	0.479	0.379	0.308	0.258	0.221	0.193	0.171	0.153
0.9864	0.847	0.619	0.470	0.371	0.300	0.251	0.215	0.187	0.166	0.148
0.9860	0.843	0.611	0.462	0.362	0.293	0.244	0.209	0.182	0.161	0.144
0.9856	0.839	0.603	0.453	0.354	0.286	0.238	0.203	0.177	0.156	0.140
0.9852	0.835	0.595	0.445	0.347	0.279	0.232	0.198	0.172	0.152	0.136
0.9848	0.831	0.588	0.437	0.339	0.272	0.226	0.193	0.168	0.148	0.133
0.9844	0.827	0.581	0.429	0.332	0.266	0.221	0.188	0.164	0.145	0.129
0.9840	0.823	0.574	0.422	0.325	0.260	0.216	0.184	0.160	0.141	0.126
0.9836	0.820	0.566	0.414	0.318	0.254	0.211	0.179	0.156	0.138	0.123
0.9832	0.816	0.560	0.407	0.312	0.249	0.206	0.175	0.152	0.134	0.120
0.9828	0.812	0.553	0.400	0.306	0.243	0.201	0.171	0.149	0.131	0.117
0.9824	0.808	0.546	0.394	0.300	0.238	0.197	0.167	0.145	0.128	0.115
0.9820	0.804	0.539	0.387	0.294	0.233	0.192	0.164	0.142	0.125	0.112
0.9816	0.801	0.533	0.381	0.288	0.228	0.188	0.160	0.139	0.123	0.110
0.9812	0.797	0.527	0.374	0.283	0.224	0.185	0.157	0.136	0.120	0.107
0.9808	0.793	0.520	0.368	0.278	0.219	0.181	0.153	0.133	0.118	0.105
0.9804	0.789	0.514	0.362	0.273	0.215	0.177	0.150	0.130	0.115	0.103
0.9800	0.786	0.508	0.357	0.268	0.211	0.174	0.147	0.128	0.113	0.101

*The formula for computing the figures in this table is as follows:

$$W_N = \frac{1 - K^{52(N-5)}}{52(N-5)(1-K)},$$ where N is years of Duration.



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K. The terms are in years of Duration, one through 10.[3] Using years is important, because it signifies the exponent on the *weekly* Ks needed for each W, according to Equation (8A). A weekly K gives much better yield curves than monthly, quarterly or annual Ks. For this reason, we assume a one-week holding period and use weekly risk-free rates. The curve that results from using such short intervals is continuous and does not require any interpolation.

With K at 0.99, the W for the third year would be computed as follows:

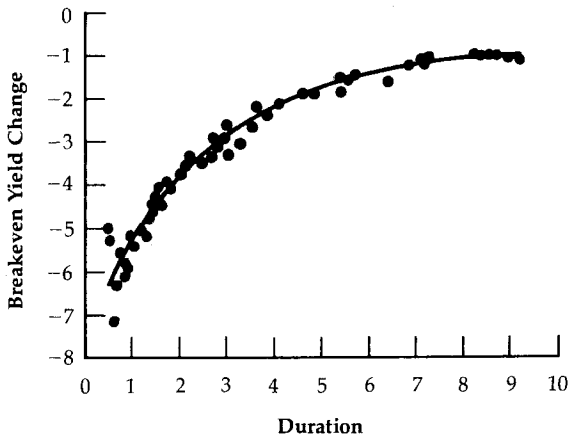
$$W_3 = \frac{1-(0.99)^{156}}{(156)(0.01)} = 0.507.$$

Table III
Product of duration and W for various values of K and duration

	Weight times duration									
	Duration									
K	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
0.9996	0.995	1.970	2.924	3.859	4.774	5.670	6.548	7.408	8.250	9.075
0.9992	0.990	1.940	2.850	3.725	4.560	5.363	6.133	6.872	7.581	8.260
0.9988	0.985	1.910	2.779	3.595	4.359	5.078	5.754	6.388	6.984	7.543
0.9984	0.980	1.882	2.711	3.473	4.170	4.814	5.406	5.950	6.451	6.911
0.9980	0.975	1.854	2.644	3.356	3.992	4.568	5.087	5.553	5.973	6.351
0.9976	0.971	1.826	2.580	3.244	3.825	4.340	4.794	5.193	5.545	5.855
0.9972	0.966	1.799	2.518	3.137	3.667	4.127	4.524	4.866	5.160	5.414
0.9968	0.961	1.772	2.457	3.035	3.518	3.929	4.275	4.568	4.814	5.021
0.9964	0.956	1.746	2.399	2.938	3.378	3.744	4.047	4.296	4.501	4.670
0.9960	0.952	1.721	2.343	2.845	3.245	3.572	3.835	4.047	4.218	4.355
0.9956	0.947	1.696	2.288	2.756	3.120	3.411	3.640	3.821	3.962	4.073
0.9952	0.942	1.672	2.236	2.671	3.002	3.260	3.460	3.613	3.730	3.819
0.9948	0.938	1.648	2.184	2.590	2.890	3.120	3.292	3.422	3.519	3.590
0.9944	0.933	1.624	2.135	2.512	2.784	2.988	3.138	3.247	3.326	3.383
0.9940	0.928	1.601	2.087	2.438	2.684	2.865	2.994	3.085	3.150	3.195
0.9936	0.924	1.578	2.041	2.367	2.589	2.749	2.860	2.937	2.989	3.025
0.9932	0.919	1.556	1.996	2.298	2.499	2.640	2.736	2.800	2.842	2.869
0.9928	0.915	1.534	1.952	2.233	2.414	2.538	2.620	2.673	2.706	2.727
0.9924	0.911	1.513	1.910	2.170	2.333	2.442	2.512	2.555	2.582	2.597
0.9920	0.906	1.492	1.869	2.110	2.257	2.352	2.411	2.447	2.467	2.477
0.9916	0.902	1.472	1.830	2.053	2.184	2.267	2.317	2.345	2.361	2.368
0.9912	0.897	1.451	1.791	1.997	2.115	2.187	2.229	2.251	2.262	2.266
0.9908	0.893	1.432	1.754	1.944	2.049	2.112	2.146	2.164	2.171	2.173
0.9904	0.889	1.412	1.718	1.894	1.986	2.041	2.069	2.082	2.086	2.086
0.9900	0.884	1.393	1.683	1.845	1.927	1.973	1.996	2.006	2.008	2.006
0.9896	0.880	1.375	1.649	1.798	1.870	1.910	1.928	1.934	1.934	1.931
0.9892	0.876	1.356	1.616	1.753	1.816	1.850	1.863	1.867	1.865	1.862
0.9888	0.872	1.338	1.584	1.709	1.765	1.793	1.803	1.804	1.801	1.797
0.9884	0.868	1.321	1.553	1.668	1.716	1.739	1.746	1.745	1.741	1.736
0.9880	0.863	1.303	1.523	1.628	1.669	1.687	1.692	1.690	1.685	1.679
0.9876	0.859	1.286	1.493	1.589	1.624	1.639	1.641	1.637	1.632	1.626
0.9872	0.855	1.270	1.465	1.552	1.581	1.593	1.592	1.588	1.582	1.575
0.9868	0.851	1.253	1.437	1.517	1.541	1.549	1.547	1.541	1.535	1.528
0.9864	0.847	1.237	1.411	1.482	1.502	1.507	1.503	1.497	1.490	1.484
0.9860	0.843	1.222	1.385	1.449	1.464	1.467	1.462	1.455	1.448	1.441
0.9856	0.839	1.206	1.359	1.417	1.428	1.429	1.423	1.416	1.408	1.402
0.9852	0.835	1.191	1.335	1.386	1.394	1.393	1.386	1.378	1.371	1.364
0.9848	0.831	1.176	1.311	1.357	1.361	1.358	1.351	1.342	1.335	1.328
0.9844	0.827	1.161	1.288	1.328	1.330	1.325	1.317	1.309	1.301	1.294
0.9840	0.823	1.147	1.265	1.301	1.300	1.294	1.285	1.276	1.269	1.262
0.9836	0.820	1.133	1.243	1.274	1.271	1.263	1.254	1.245	1.238	1.231
0.9832	0.816	1.119	1.222	1.248	1.243	1.235	1.225	1.216	1.208	1.202
0.9828	0.812	1.105	1.201	1.223	1.216	1.207	1.197	1.188	1.180	1.174
0.9824	0.808	1.092	1.181	1.199	1.190	1.180	1.170	1.161	1.154	1.147
0.9820	0.804	1.079	1.161	1.176	1.165	1.155	1.145	1.136	1.128	1.122
0.9816	0.801	1.066	1.142	1.154	1.141	1.131	1.120	1.111	1.104	1.098
0.9812	0.797	1.053	1.123	1.132	1.118	1.107	1.096	1.087	1.080	1.074
0.9808	0.793	1.041	1.105	1.111	1.096	1.085	1.074	1.065	1.058	1.052
0.9804	0.789	1.029	1.087	1.091	1.075	1.063	1.052	1.043	1.036	1.030
0.9800	0.786	1.017	1.070	1.071	1.054	1.042	1.031	1.022	1.016	1.010

Chart C

Fitted Breakeven Values (April 14, 1980)



Yield Curve Implied by Breakeven Fit (April 14, 1980)

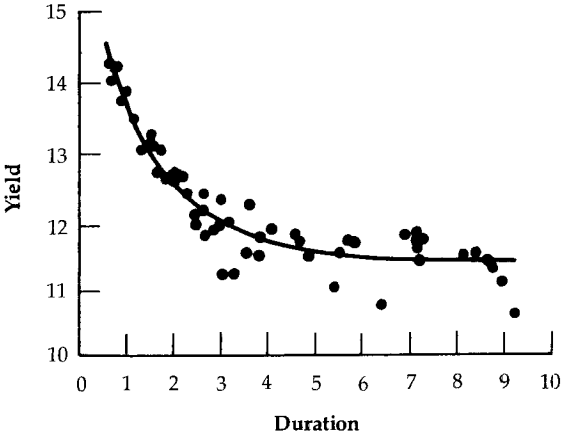
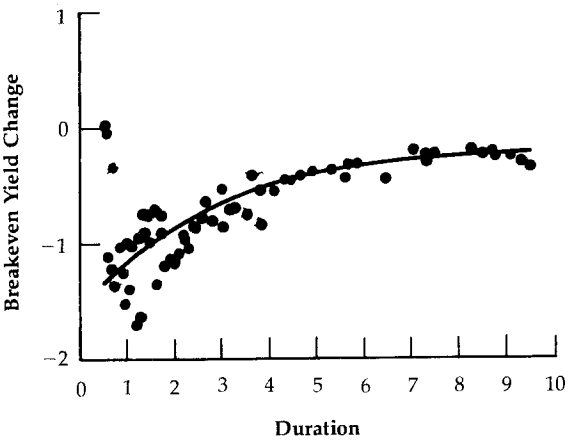


Chart D

Fitted Breakeven Values (April 29, 1980)



Yield Curve Implied by Breakeven Fit (April 29, 1980)

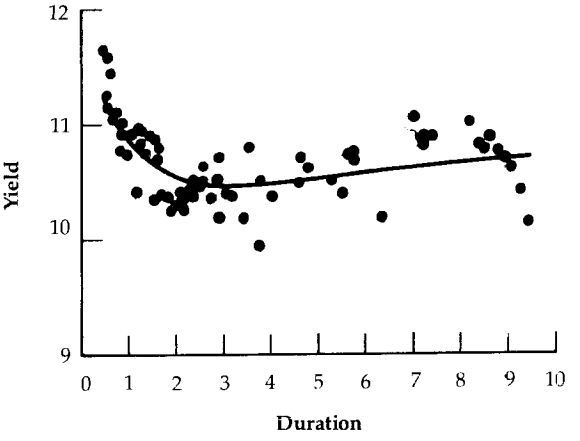
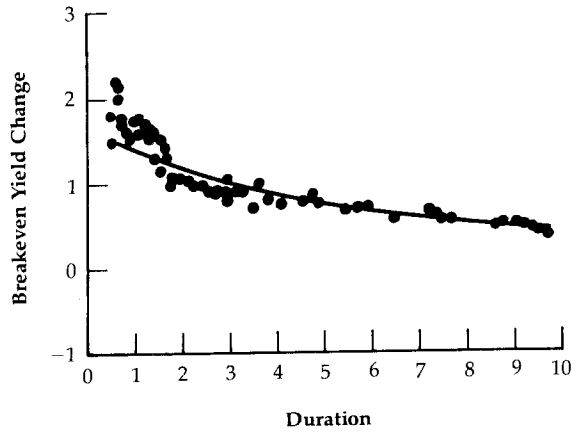


Chart E

Fitted Breakeven Values (May 29, 1980)



Yield Curve Implied by Breakeven Fit (May 29, 1980)

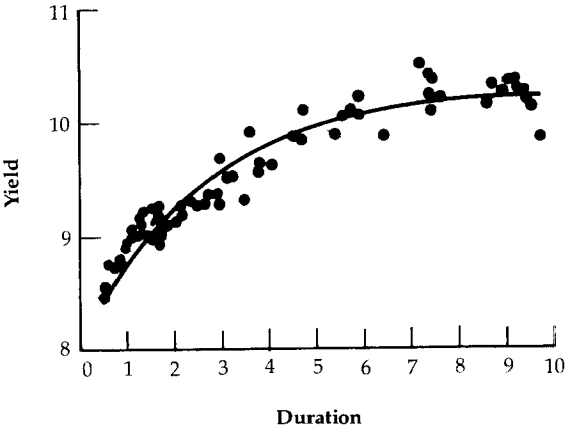
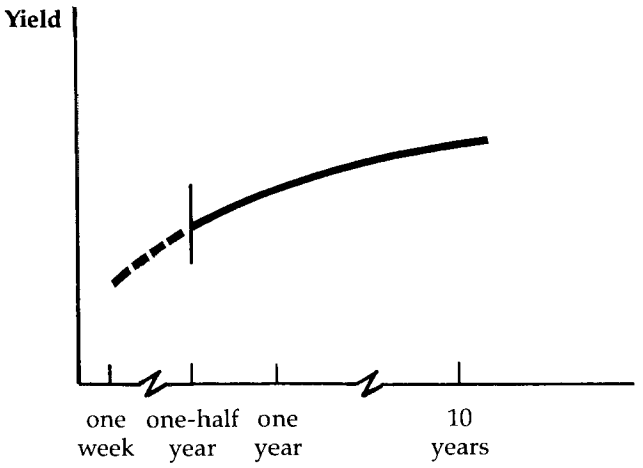


Chart F – The yield curve and the one-week rate



Thus the average expected one-week change in the one-week rate over the next three years is 0.507 times as large as the expected change in this rate next week. However, because some of the curves are easier to fit when the starting point for exponentiating K is put off beyond the first week, we assume the exponential starts working at six months. For the above example, the exponent on K at the end of three years would be 130 – the number of weeks in 2.5 years – instead of 156, giving:

$$W_3 = \frac{1-(0.99)^{130}}{(130)(0.01)} = 0.561,$$

which is the number shown in Table II for a Duration of three and a K of 0.99.

Table III shows $D_N W_N$ – the active part of Equation (13). Whereas the sign on $\hat{BYC}$ determines whether the curve is positive or inverted, $D_N W_N$ determines the rate at which it rises or falls, or rises and falls. For high values of K, this product declines uniformly, giving either a positive or inverse yield curve. But for lower Ks – for example, 0.9848 – it rises and then falls, giving a U-shaped or a humped yield curve for negative and positive values of $\hat{BYC}$, respectively.

Charts C through E illustrate the variety of yield curves that can be fitted with this method.

In each case, the BYC curve has an exponential shape; yet the yield curves are different. Chart C is a fairly typical inverted yield curve; Chart D is a U-shaped curve; Chart E is a normally rising curve.

Note that the scatter in the BYC is much greater at the short end than at the long, and that the difference in scatter disappears and even reverses in the yield curve. The reason is that prices move less at the short end for a given yield move. Breakeven comes when the bond price changes by the amount needed to neutralize the spread between the bond and the bill. For short-term bonds, there are a lot of basis points in a small price move and in a small difference from the price moves of nearby bonds. In the transformation to yield curves, the effect works the other way – the small price impact of the yield disparities at the short end causing them to bunch together when plotted on the yield curve.

Equation (11) indicates why an error in the BYC counts more at the long end of the yield curve. The bond yield is merely the sum of the bill yield and the annualized product of Duration and the BYC. An error in the BYC hurts more at the long end, where it is multiplied by a larger Duration. In searching for the three parameters that give the best fit, it makes sense to give more weight to the longer term BYCs than to the shorter ones.

The integrity of the yield curve

The yield curve derived from our model depends on three variables – Y_B, the one-period rate with which the bond returns are compared; K, the decay rate of forecast yield changes for one-period intervals in the future; and $\Delta Y_{B,1}$, the forecast change of the one-period rate for one period. Chart F shows the yield curve in relation to the one-week rate. The curve starts at the half-year rate because the shorter rates are influenced by money-market pressures not accounted for in our model (see the section on the liquidity premium below.) Although not part of the yield curve, the short rate is uniquely related to it. It influences the pattern of BYCs.

One part of the fitting process finds the short rate whose BYCs give the best fit. The search usually converges on a rate that is very close to the federal funds rate, which itself plays no role in the fit. Chart G plots the two rates daily since the beginning of 1980. The correlation between the two short rates, one real and the other hypothetical, is remarkable in view of the way the hypothetical rate is obtained. It suggests a strong linkage between the shape of the total yield curve and the short-term rate, which, as noted, is not part of the curve.

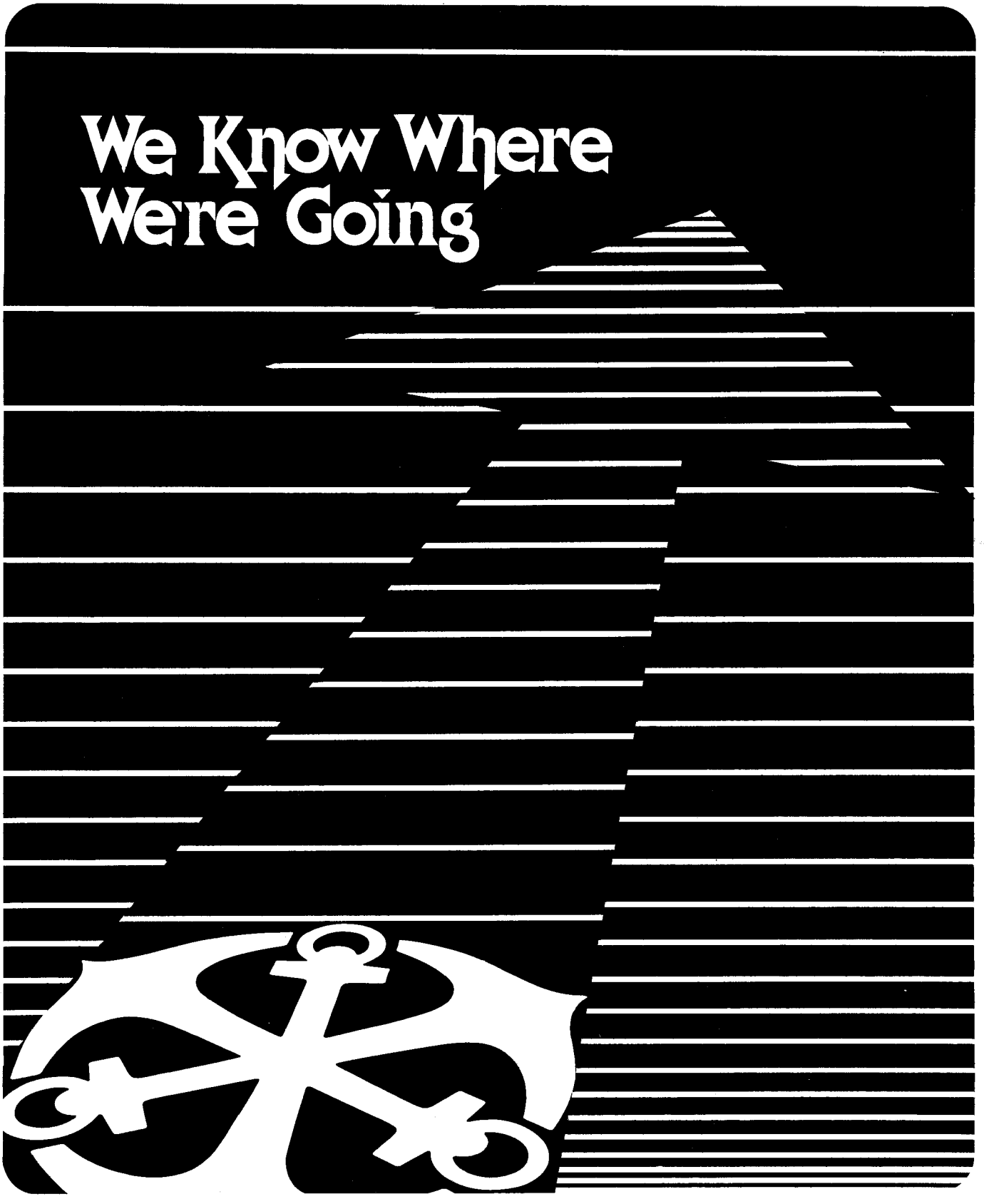
The shape of the curve should be viewed as a single concept, not as a concatenation of many fragmentary expectations. Using Hicks' coefficient of expectations to impose a single integrating concept is a key source of the model's strength. It allows a fairly simple curve-fitting routine in place of the complex number crunches people usually bring to this problem. This unified approach works notwithstanding the vagaries of markets and market quotations, suggesting that the curve really does depend on elementary forces even if no one player can put his finger on them.

The plot of the Ks in Chart H covers a period when some of the most violent shifts of the yield curve occurred. Yet the Ks (estimates of Hicks' coefficient) make a fairly smooth curve, except for the spikes in April, a time of transition between the earlier inverted and later positive slope. (See the appendix for a discussion of relative yield volatility.)

Yield spreads

Turnover costs, ignored until now, are part of the comparison between the returns on a bond and the locked-in yield on a bill. Buying the bond at the offer and selling it at the bid price lowers its expected round-trip return in relation to the bill's and makes people look for compensatory yield up front. The wider the dealer spread, and the more problematic the market at the time of sale, the less appealing the bond over the bill and the greater compensatory bond yield required. For the arbitrageur, as distinct from the cash buyer, the cost of carrying the bond also affects this decision. Being able to carry Treasuries on repo while having to finance corporate bonds with bank loans lowers the relative size of the sweetener required on the former.

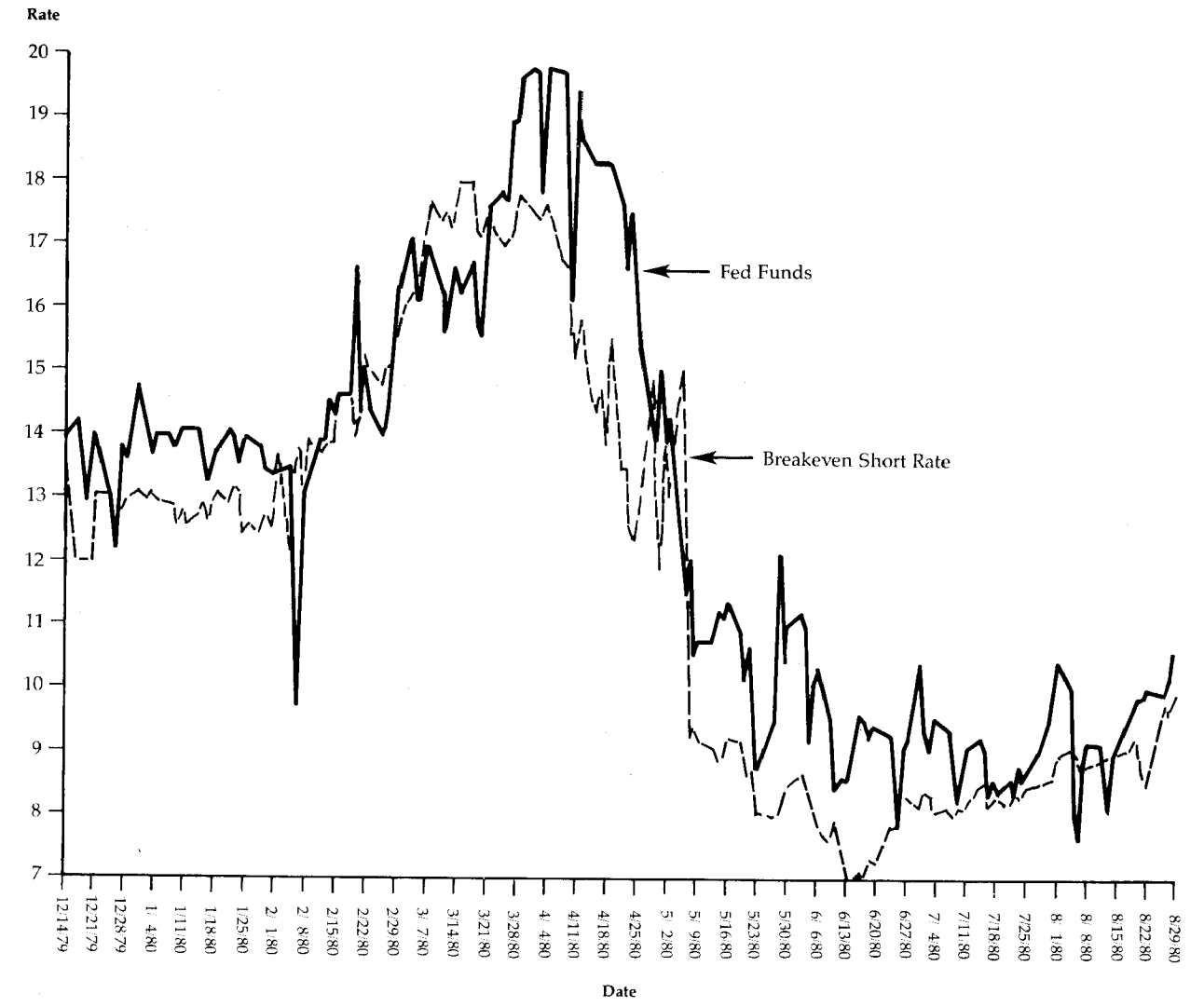
According to our model, the market sets bond yields at levels where their expected round-trip returns equal the bill yield. Turnover costs and the risk of dealer spreads' widening bear on the size and variation of intersectoral spreads. For example, in the tax-exempt markets, the yield curve slopes upward virtually all the time, even when the



It's at times like these that Expertise Counts

OLD MUTUAL

Chart G Fed Funds vs. Fitted Breakeven Short Rate



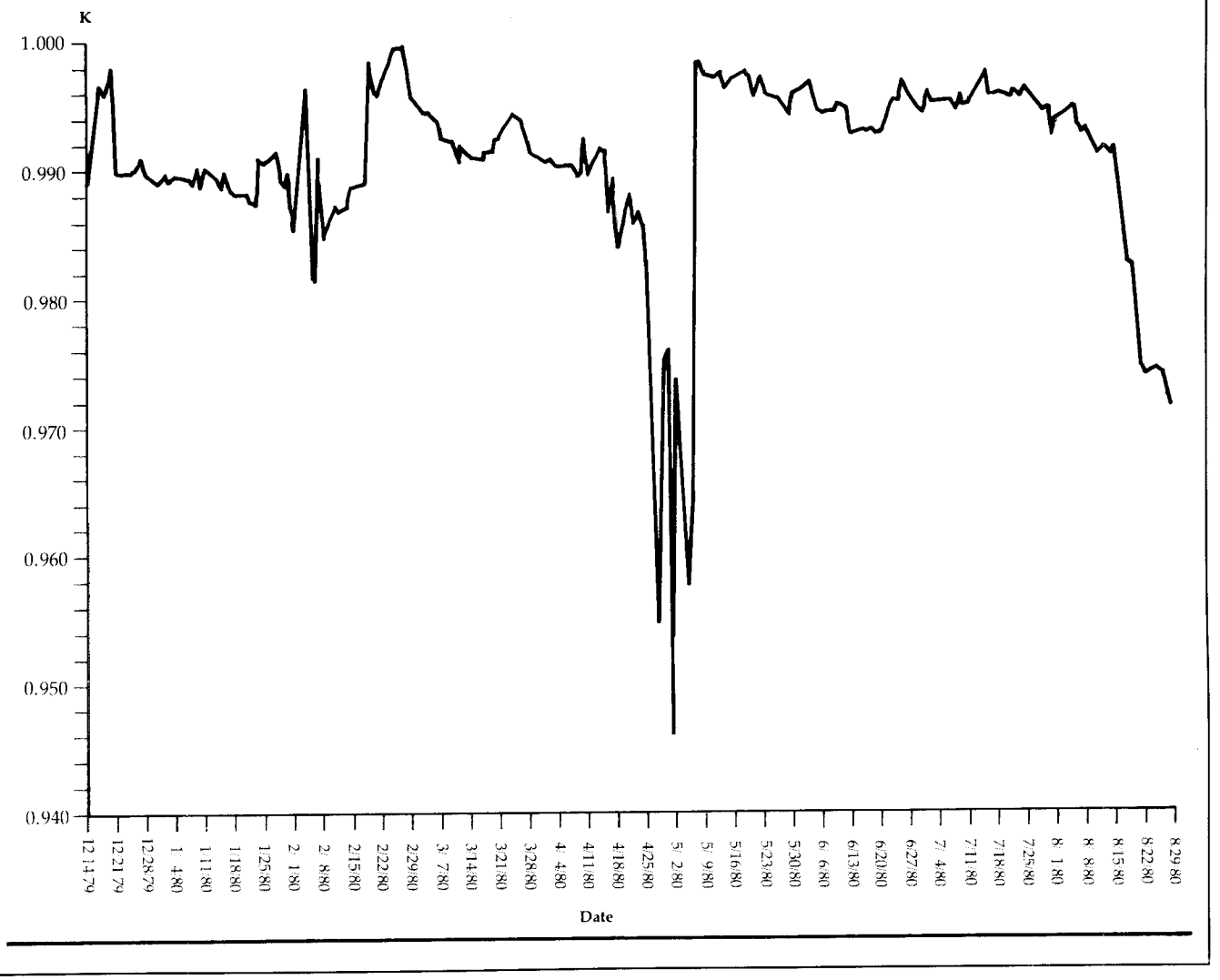
Treasury curve is inverted. When both curves are positive, the tax-exempt one is steeper. The most important reason for the difference is turnover costs. Buying a long tax exempt bond for a short holding period can be an expensive alternative to locking in a short rate.

Another example is the variation in the spreads between U.S. Agencies and Treasuries in different market environments. A few years ago, the average spread between the two sectors was roughly 15 to 20 basis points. At the peak yields early in 1980, it had widened to about 75 basis points (see Chart I). Some ascribe this change to a decline in the quality of Agency bonds or to increased aversion to the original quality, but neither explanation makes sense. Instead, the larger spread was probably due to the wider spread between bid and asked prices and to generally thinner markets. It is harder to do a round trip with Agencies than with Treasuries in any market – an important source of even their normal spread – but harder still in a bear market. The problem is liquidity, not credit quality.

Similar examples abound with Euro and Yankee bonds, whose spreads against Treasuries also opened during the bear market, again as a result of liquidity rather than quality. In fact, as Chart J shows, these bonds were harder hit during the bear market than comparable quality, but more liquid, industrials. The spread between representative Yankee and corporate bonds, having risen almost 100 basis points by early spring 1980, the nadir of the market, returned to earlier levels when the market came back. It would be very hard to ascribe this swing to a change in perceived quality.

It may seem out of place to consider the spread between two long bonds in a study of the yield curve. But both belong to some yield curve and have to break even against some short-term yield. A falloff in liquidity raises the starting yield needed to equate expected return with the short yield in the same sector, or with whatever short yield the bond buyer considers an alternative to investing. Much of what passes for changing risk differentials is really this liquidity effect.

Chart H Values of K vs. Time



Liquidity premium

Over the last several years, the yield curve has as often as not been inverted over its full length, starting with the six-month term. Below this point, it is almost always positive. As shown in Chart K, the three-month bill rate was below the six-month rate even when the latter exceeded the one-year rate. Chart L shows the same for CDs, except for a brief inversion early last year.

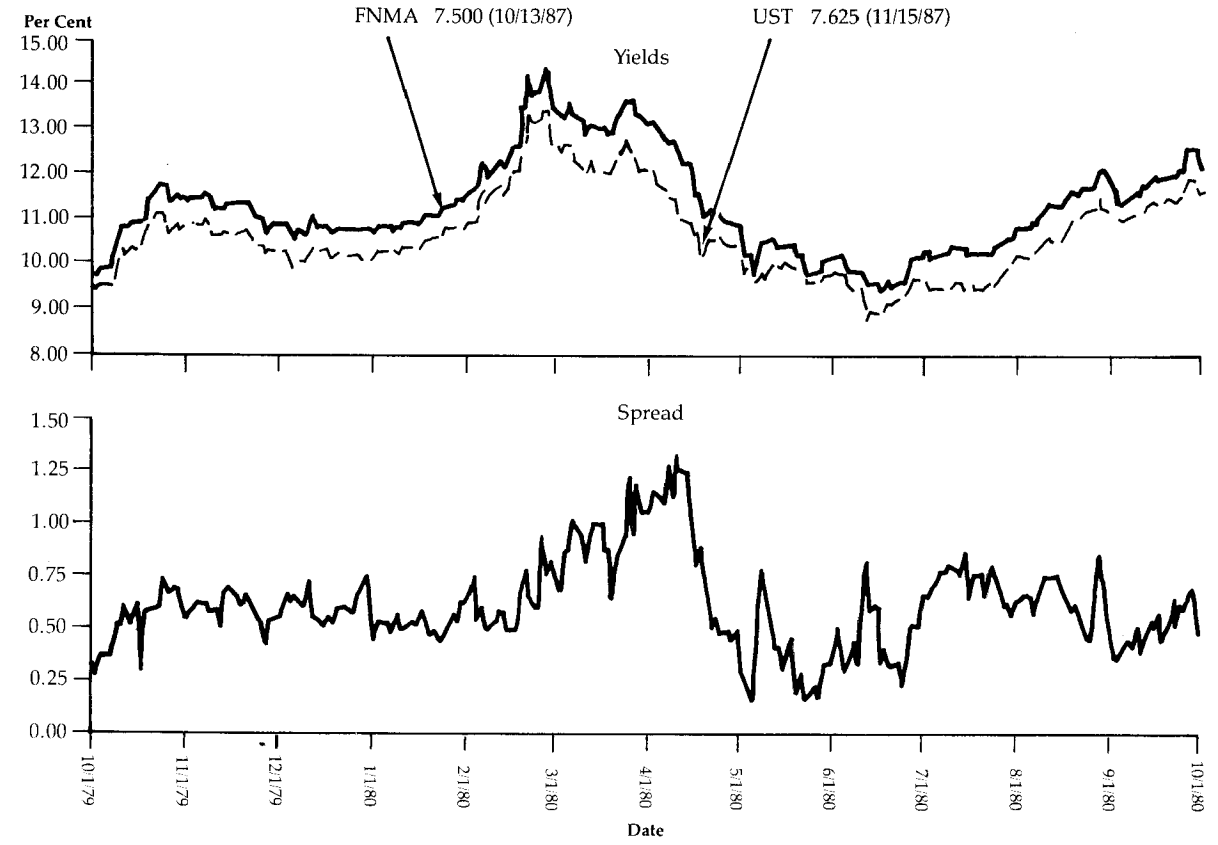
This strong positive bias at the short end of the curve is due to liquidity preference and, in our opinion, is unrelated to the greater price volatility of longer bonds. In fact, the longer short-term securities – bills, CDs and Bankers' Acceptances (BAs) – for which there are broad public markets, tend to be *more* liquid than the shorter-term money-market securities, commercial paper and repos, which experience little secondary trading.

The money market is really a market for money, rather than securities. Those who buy five-day commercial paper are really selling money, the securities merely documenting the money transfers.The buyers know in advance when they need the money and try to match terms to these needs. Liquidity comes from this foreknowledge, not from being able to sell longer securities before they mature. The issuers, on the other hand, are not looking for credit so much as they are managing their cash flow with fewer liquid assets than they would otherwise need.

Six-month securities, in contrast, are bought either as short-term investments – for example, by people anticipating a fall in the long market – or for liquid reserves, but usually not to match a known cash need. In this case, the sellers are getting short-term credit.

The upward slope at the short end of the yield curve reflects imbalances between supply and demand in these two sectors. There is probably an excess demand for very short-term securities, from people warehousing cash, and an excess supply of six-month bills, CDs and BAs, reflecting the short-term borrowings of government and banks. These longer issuers show little interest in lowering borrowing costs by selling into the lower yielding sector. For banks, it would add to the refinancing risk, and for the government, it would complicate monetary policy by adding to the stock of money substitutes. In any case, they

Chart I Two-Bond Spread History



do not do it enough to flatten the yield curve. Nor do the cash warehousers extend for more yield, because they would either lose the risk-free liability match or lengthen their average liquid asset holding at the expense of higher yielding opportunities. The difference in the function of the two markets makes the securities less interchangeable than at other parts of the curve, and it is a barrier to arbitrage in the way transit costs shield two distant markets from the full effect of competition.

The undersupply at the short end and over-supply at six months would be greater still, and the curve more positively sloped, were it not for the term repo. Arbitrageurs borrow at the repo rate for the shorter terms (like those of commercial paper) to carry the longer ones. In the process, they increase the supply of short-term and the demand for long-term (six-month) securities, narrowing the natural gap just noted. But they do not eliminate it. Chart M shows why.

The yield curve for repos is higher than the one for bills by an amount that measures the liquidity premium. (This point holds better for CDs than for Treasury bills, which yield less than their repos for reasons other than liquidity.) In the chart, the arbitrageur buys a bill with N months to maturity and does a repo for T months. At this point, the repo cost equals the bill yield and the carry is therefore zero. If he did a shorter repo, his carry would be positive, and if a longer one, negative. After T months, he must refinance or sell the bill, at potentially higher rates. The months remaining on the bill at this point are called the tail. The longer the tail, the greater the potential price change on the bill or, what comes to the same thing, the longer the period of refinancing risk.

Assuming for a moment that arbitrageurs are content with a zero carry (or any fixed carry), a steeper yield curve or a smaller liquidity premium allows a shorter tail and therefore less risk. If either of these got out of line, the curve too steep or the premium too small, the demand for bills and the supply of repos would rise until the curve and premium were in balance. Similarly, an increase in the liquidity premium would lengthen the required tail, raise the risk and consequently lower the demand for bills and the supply of repos. In this case, the curve would get steeper.

The point is that the short-term market is held in check by (1) the perceived risk for a given tail, (2) the outlook for yield changes and (3) the liquidity premium (which, as suggested in the chart, increases with term). Arbitrageurs would have to be very bullish to buy enough bills to flatten the yield curve and accept the liquidity premium as a

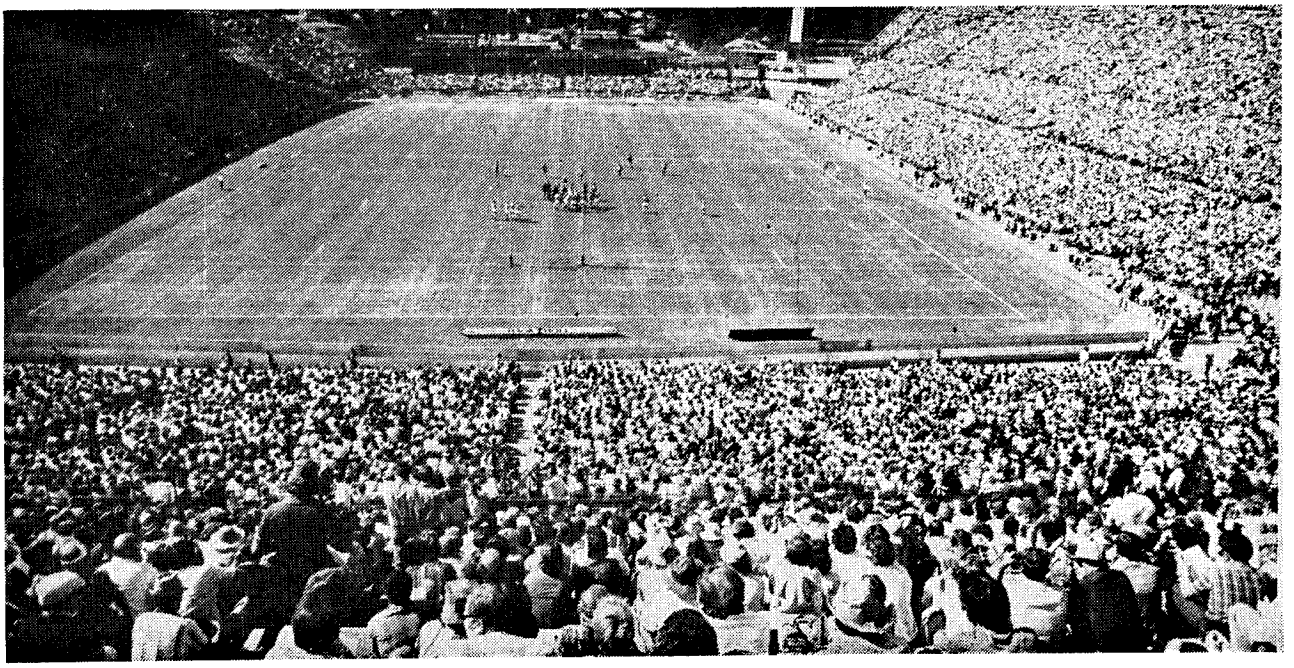


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Chart J Two-Bond Spread History

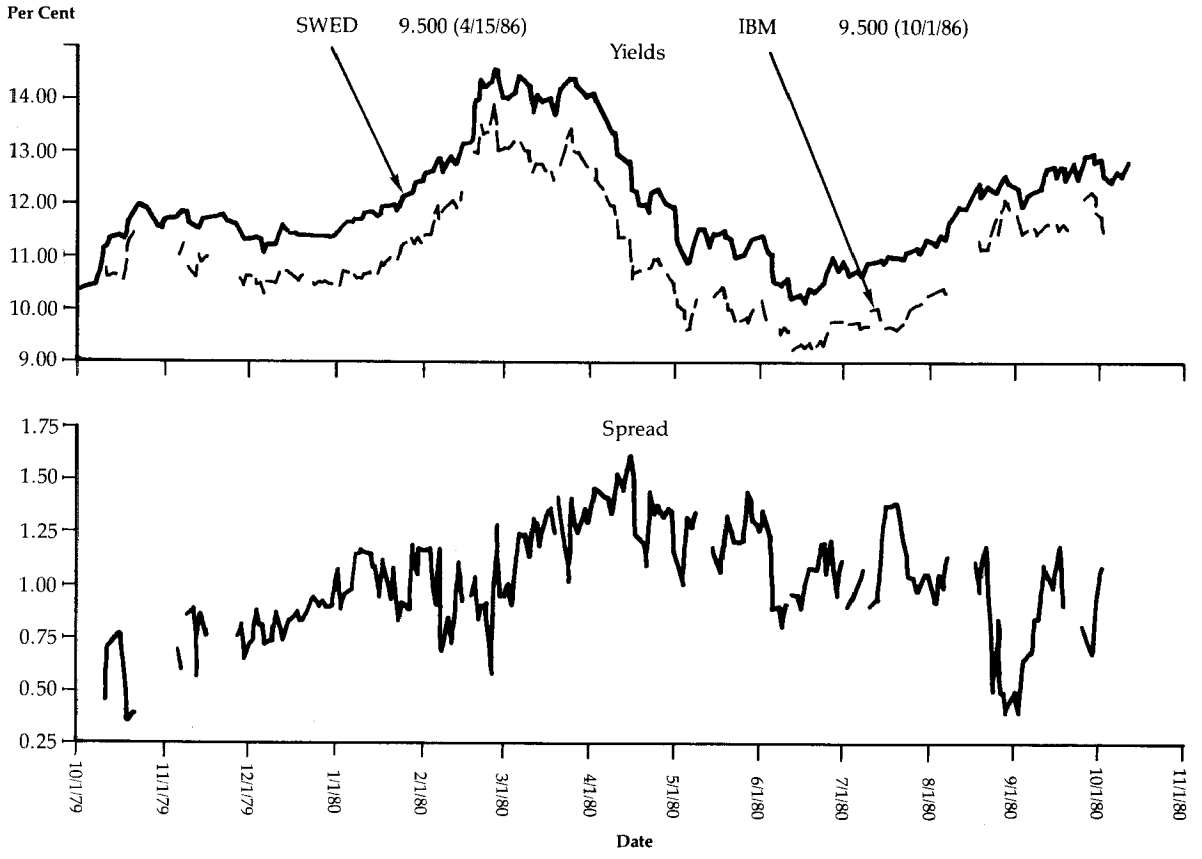


Chart K Yield Spreads Between 12-Month and Six-Month and Six-Month and Three-Month T-Bills

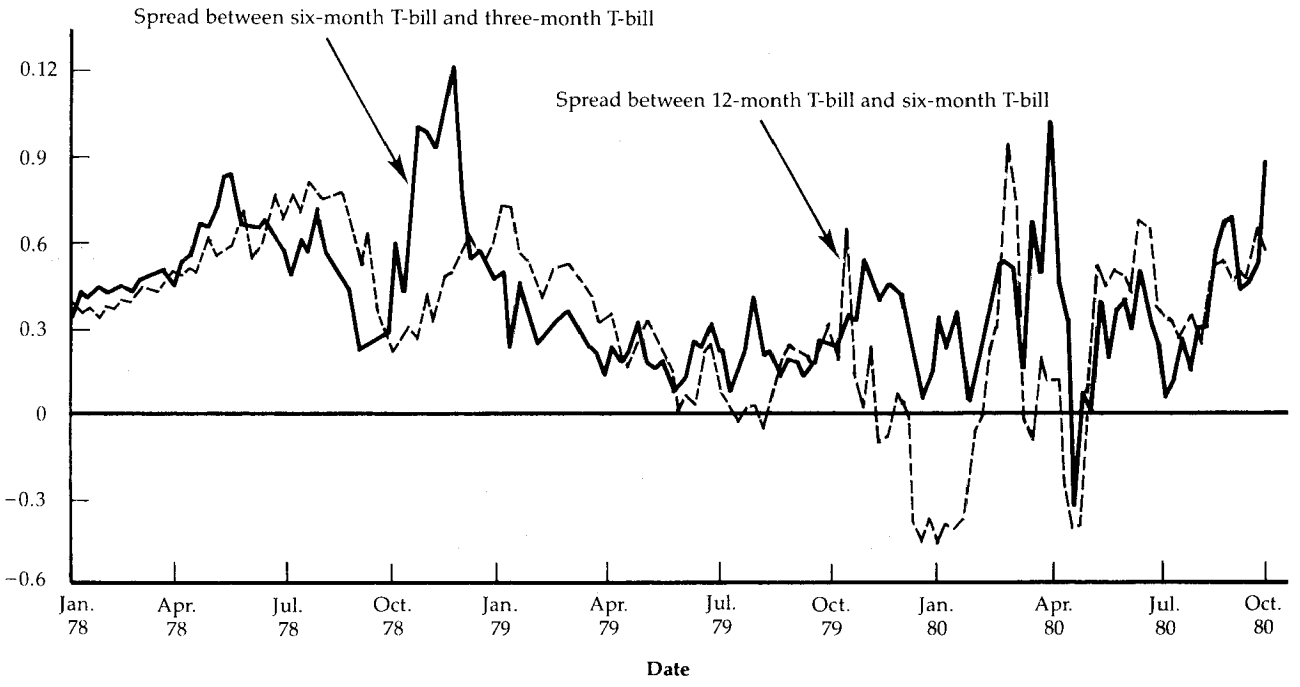


Chart L Yield Spreads Between 12-Month and Six-Month and Six-Month and Three-Month CDs

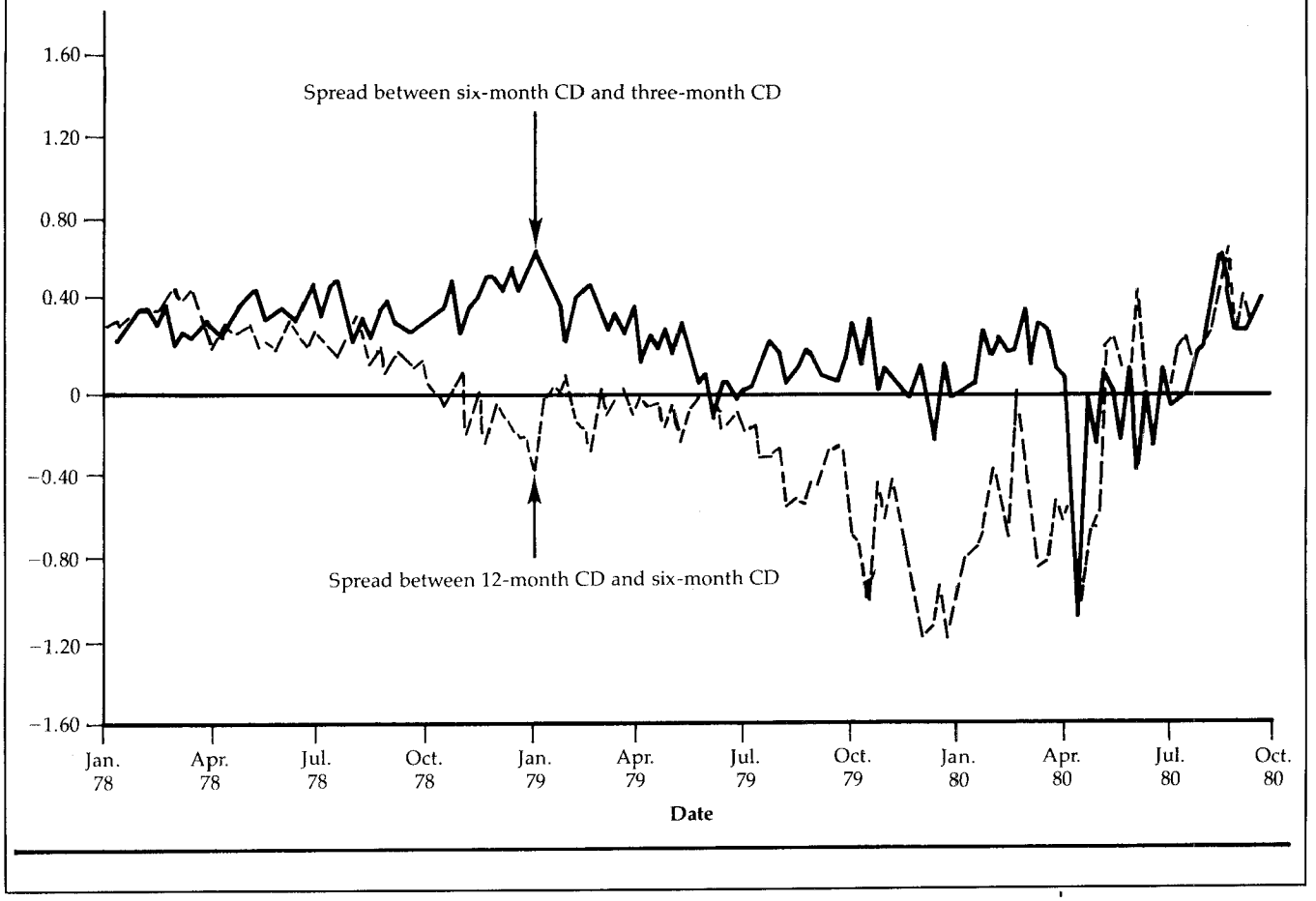
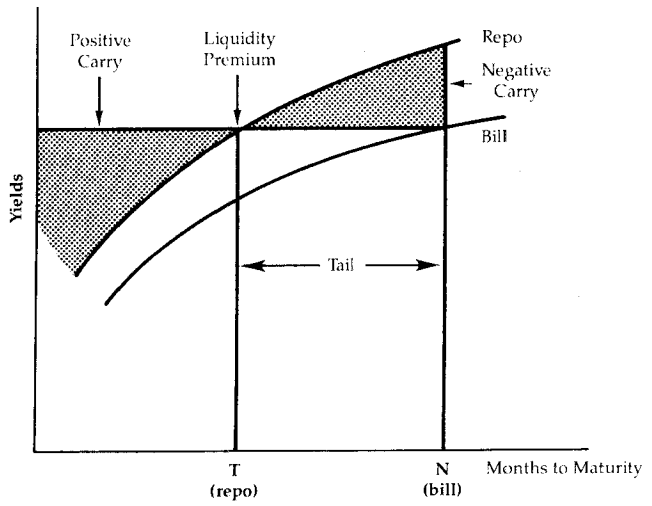


Chart M The Repo and Bill Rates



negative carry. Barring that, there is no market force that would entirely offset the imbalance between supply and demand described earlier.

Footnotes

1. John R. Hicks, *Value and Capital* (London: Oxford University Press, 1939), p. 262.
2. Equating BYCs with *forecast* changes implies that the market will accept the same returns on securities with different risks. But most people are not content with a bond's merely breaking even against a bill; they want something extra for risk. For this reason, equating BYCs with expected yield changes is not quite right. Yet fitting them as if they were expected change works quite well. The reason, I think, lies in the way risk works in short-term holdings.

   One widely used measure of risk is dealer spread. It is usually greater for longer bonds than for shorter ones. In today's markets, aside from a very few active issues, the typical spreads are 1/16 to 1/8 at the short end of the curve and 1/8 to 1/4 at the long end. But the yield equivalent of the spread clearly declines with increasing term. At a Duration of 2.0, 1/8 is worth 6.25 basis points; at a Duration of 10, it is worth 1.25 basis points. Therefore, even with the rise in dollar spreads with increasing term, the yield equivalent of the spread falls.

   Allowing for dealer's spread is one way to take care of risk. It makes the expected change in yield more favorable to the investor than the breakeven yield change, at least by enough to cover the dealer's spread. For example, in the case of an inverted yield curve, the expected drop in yields would exceed the breakeven by the amount needed to cover turnover cost. Further, the investor can imagine an added cost on longer bonds because of their extra market risk. The cost would make the expected yield changes still more favorable than the breakeven changes. But the yield equivalent of these added costs would also fall with increasing term as a result of the Duration effect noted above. Therefore, replacing the BYCs with expected changes, which allow for turnover cost and risk, would give a very similar curve to the one fitted in the present model.

3. Using the lable Duration, instead of maturity, has no effect on the computation, but only on how one interprets the results. Duration is a better measure of volatility than is term to maturity. But it is also a better measure of term because it corresponds more closely, albeit not perfectly, to distinctions people actually make. The five years between a 25 and 30-year bond are not the same as those between a five and 10-year bond. At current interest rates, the more distant five calendar years add very little to Duration, while the nearer five add about two years.

   What one thinks will happen 21 years out is not very different from events forecast 20 years out. Closer in, a year could make a lot of difference. Put another way, the units of future time between which people draw sharp distinctions get much larger as one looks further into the future. In looking backward, people also tend to block out larger chunks of time the further back they go. Duration gives the same effect; there are more calendar years between Durations 9 and 10 than between Durations 1 and 2.

APPENDIX

Relative yield volatility

Table AI shows the correlation and regression coefficients of various long yields in relation to the one-year yield. Both coefficients are lower for the longer yields, the second much more so than the first, which makes sense within the framework of the model. Further, the correlation coefficients are larger for longer changes, say quarterly instead of weekly.

Table A1
Correlation and regression coefficients of various long yields in relation to the one-year yield*

	Correlation coefficients			Regression coefficients		
On one-year yield	Weekly change	Monthly change	Quarterly change	Weekly change	Monthly change	Quarterly change
5-year	0.901	0.942	0.960	0.625	0.598	0.591
10-year	0.841	0.884	0.911	0.451	0.426	0.430
20-year	0.795	0.847	0.887	0.369	0.359	0.371

*Constant maturity data from January 1975 through August 1980. The regressions have the following forms: $\Delta Y_L = a+b\Delta Y_1+e$, where ΔY_L is the respective change in the five, 10 and 20-year yield; ΔY_1 is the change in the one-year yield; and b is the regression coefficient. a and e, the intercept and residual, are not shown.

Chart AA Expected Changes in Short Yields Over Time and Near-Term Changes in Different Long Yields

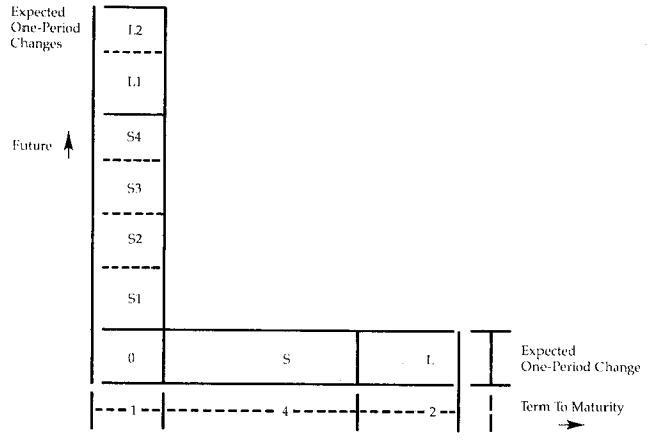


Chart AA shows how the same expected changes in future short-term yields make up the overlapping parts of expected changes in long yields. The expected yield changes over the first five years determine all the expected change in the five-year yield, as well as the first five years' worth of the longer yields.

This year's expected one-period change in the one-period yield is denoted by the box marked "0." The next four expected one-period changes in the one-period yields are denoted by S1 through S4. The average of these five expected changes equals the expected one-period change of the five-period yield, S+0. The remaining expected changes, L1 and L2, merge with the first five for the expected change in the seven-year period rate, 0+S+L. This is a major reason why the correlations in Table AI remain high as terms increase. The five-year (0+S) part of the change in the seven-year yield (0+S+L) is perfectly correlated with the change in the five-year yield.

In addition, the remaining part, L, is correlated with 0+S because of the way L1 and L2 depend on the first five one-period changes. According to the model, $S1 = 0K$; $S2 = 0K^2$ and $L2 = 0K^6$, K being the coefficient of expectation. If K were the same every day, K^6 also would be the same, and L2 would be a constant multiple of and, while smaller, perfectly correlated with it.

In this case, the yield changes on bonds with different terms would be perfectly correlated, differing by a constant multiple. In contrast, the regression coefficients would be smaller on longer bonds, since the exponent on K would be larger. But K is not constant from day to day, and as it changes, K^2, K^3 and so on, all change by different amounts, the differences growing with the size of the exponent. For example, if K changes from 0.7 to 0.8, K^2 changes from 0.49 to 0.64, and K^{10} from 0.028 to 0.107. A regression over time cuts across yield curves with different Ks, each of which implies a relative change in the long versus the short rate. The regression captures roughly the average K.

The correlations are higher for quarterly changes than for monthly or weekly ones and for monthly than for weekly changes because the market does not move as smoothly as the theory assumes. Over short periods, market imbalances get in the way of logical relationships. Over longer periods, they wash out. This effect is much stronger when it is a question of things moving together (correlation), where random shocks make it seem as if they do not, than when it

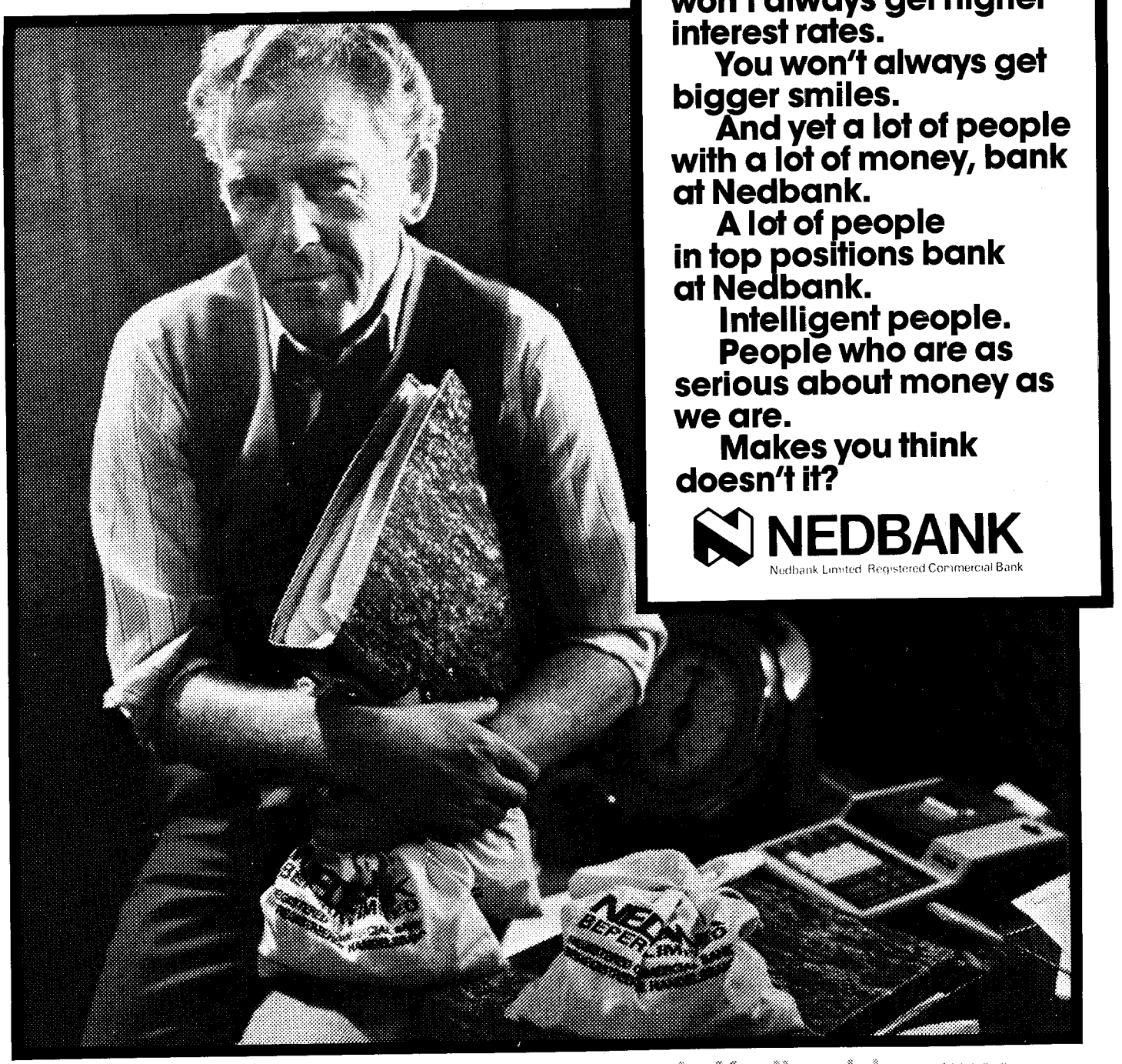


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Table A11
Ratios of the Ws for each duration and the W for unit duration

	Duration									
K	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0
0.9996	1.000	0.985	0.975	0.965	0.955	0.945	0.935	0.926	0.917	0.907
0.9992	1.000	0.970	0.950	0.931	0.912	0.894	0.876	0.859	0.842	0.826
0.9988	1.000	0.955	0.926	0.899	0.872	0.846	0.822	0.799	0.776	0.754
0.9984	1.000	0.941	0.904	0.868	0.834	0.802	0.772	0.744	0.717	0.691
0.9980	1.000	0.927	0.881	0.839	0.798	0.761	0.727	0.694	0.664	0.635
0.9976	1.000	0.913	0.860	0.811	0.765	0.723	0.685	0.649	0.616	0.585
0.9972	1.000	0.899	0.839	0.784	0.733	0.688	0.646	0.608	0.573	0.541
0.9968	1.000	0.886	0.819	0.759	0.704	0.655	0.611	0.571	0.535	0.502
0.9964	1.000	0.873	0.800	0.735	0.676	0.624	0.578	0.537	0.500	0.467
0.9960	1.000	0.860	0.781	0.711	0.649	0.595	0.548	0.506	0.469	0.436
0.9956	1.000	0.848	0.763	0.689	0.624	0.568	0.520	0.478	0.440	0.407
0.9952	1.000	0.836	0.745	0.668	0.600	0.543	0.494	0.452	0.414	0.382
0.9948	1.000	0.824	0.728	0.647	0.578	0.520	0.470	0.428	0.391	0.359
0.9944	1.000	0.812	0.712	0.628	0.557	0.498	0.448	0.406	0.370	0.338
0.9940	1.000	0.800	0.696	0.609	0.537	0.477	0.428	0.386	0.350	0.320
0.9936	1.000	0.789	0.680	0.592	0.518	0.458	0.409	0.367	0.332	0.302
0.9932	1.000	0.778	0.665	0.575	0.500	0.440	0.391	0.350	0.316	0.287
0.9928	1.000	0.767	0.651	0.558	0.483	0.423	0.374	0.334	0.301	0.273
0.9924	1.000	0.757	0.637	0.543	0.467	0.407	0.359	0.319	0.287	0.260
0.9920	1.000	0.746	0.623	0.528	0.451	0.392	0.344	0.306	0.274	0.248
0.9916	1.000	0.736	0.610	0.513	0.437	0.378	0.331	0.293	0.262	0.237
0.9912	1.000	0.726	0.597	0.499	0.423	0.365	0.318	0.281	0.251	0.227
0.9908	1.000	0.716	0.585	0.486	0.410	0.352	0.307	0.270	0.241	0.217
0.9904	1.000	0.706	0.573	0.473	0.397	0.340	0.296	0.260	0.232	0.209
0.9900	1.000	0.697	0.561	0.461	0.385	0.329	0.285	0.251	0.223	0.201
0.9896	1.000	0.687	0.550	0.449	0.374	0.318	0.275	0.242	0.215	0.193
0.9892	1.000	0.678	0.539	0.438	0.363	0.308	0.266	0.233	0.207	0.186
0.9888	1.000	0.669	0.528	0.427	0.353	0.299	0.258	0.226	0.200	0.180
0.9884	1.000	0.660	0.518	0.417	0.343	0.290	0.249	0.218	0.193	0.174
0.9880	1.000	0.652	0.508	0.407	0.334	0.281	0.242	0.211	0.187	0.168
0.9876	1.000	0.643	0.498	0.397	0.325	0.273	0.234	0.205	0.181	0.163
0.9872	1.000	0.635	0.488	0.388	0.316	0.265	0.227	0.198	0.176	0.158
0.9868	1.000	0.627	0.479	0.379	0.308	0.258	0.221	0.193	0.171	0.153
0.9864	1.000	0.619	0.470	0.371	0.300	0.251	0.215	0.187	0.166	0.148
0.9860	1.000	0.611	0.462	0.362	0.293	0.244	0.209	0.182	0.161	0.144
0.9856	1.000	0.603	0.453	0.354	0.286	0.238	0.203	0.177	0.156	0.140
0.9852	1.000	0.595	0.445	0.347	0.279	0.232	0.198	0.172	0.152	0.136
0.9848	1.000	0.588	0.437	0.339	0.272	0.226	0.193	0.168	0.148	0.133
0.9844	1.000	0.581	0.429	0.332	0.266	0.221	0.188	0.164	0.145	0.129
0.9840	1.000	0.574	0.422	0.325	0.260	0.216	0.184	0.160	0.141	0.126
0.9836	1.000	0.566	0.414	0.318	0.254	0.211	0.179	0.156	0.138	0.123
0.9832	1.000	0.560	0.407	0.312	0.249	0.206	0.175	0.152	0.134	0.120
0.9828	1.000	0.553	0.400	0.306	0.243	0.201	0.171	0.149	0.131	0.117
0.9824	1.000	0.546	0.394	0.300	0.238	0.197	0.167	0.145	0.128	0.115
0.9820	1.000	0.539	0.387	0.294	0.233	0.192	0.164	0.142	0.125	0.112
0.9816	1.000	0.533	0.381	0.288	0.228	0.188	0.160	0.139	0.123	0.110
0.9812	1.000	0.527	0.374	0.283	0.244	0.185	0.157	0.136	0.120	0.107
0.9808	1.000	0.520	0.368	0.278	0.219	0.181	0.153	0.133	0.118	0.105
0.9804	1.000	0.514	0.362	0.273	0.215	0.177	0.150	0.130	0.115	0.103
0.9800	1.000	0.508	0.357	0.268	0.211	0.174	0.147	0.128	0.113	0.101

concerns how much one thing moves relative to another (regression). In this case, the shocks can be on either side of the true number and roughly wash out, leaving the regression coefficient more or less intact. In any case, the errors in the measured relationship are not helped much by allowing time for them to wash out.

Table A111
Comparison between past volatility of long yields vs. short yields and the counterparts predicted by the model*

Term (years)	Average yields 1975-1980	Estimated average duration	Regression coefficients	$W_D W_1$ K = 0,9920)
(1)	(2)	(3)	(4)	(5)
2	8,14	1,80	0,840	0,867
3	8,21	2,60	0,735	0,753
5	8,36	4,00	0,598	0,596
7	8,46	5,20	0,494	0,501
10	8,55	6,65	0,426	0,416
20	8,67	9,45	0,359	0,305

*The regression coefficients (Column 4) were obtained by regressing each long-term yield on the one-year yield, using constant maturity data for the period January 1975 to August 1980, published by the Federal Reserve Board. Column 5 gives the predicted coefficients interpolated from Table AII (for K = 0,9920) after translating the terms to maturity (Column 1) into Duration (Column 3) at the average yields shown in Column 2.

Table AIV
Regression coefficients of longer yields regressed on one-year yields over periods of varying length*

Term to maturity (years)	Jan 1975 – Aug 1980	Sept 1978 – Sept 1980	Jan 1970 – Sept 1980
2	0,840	0,833	0,839
3	0,735	0,725	0,732
5	0,598	0,597	0,599
7	0,494	0,492	0,491
10	0,426	0,421	0,415
20	0,359	0,358	0,345

*Constant maturity data of Federal Reserve Board.

The most interesting part of Table AI is the fall in the regression coefficients as the terms of the yield changes rise. In a rough way, these figures coincide with the relative Ws shown in Table AII, obtained by dividing the Ws in Table II by the ones for unit Duration. The comparison is rough for several reasons. First, whereas the Ws are given for units of Duration, the regression coefficients in Table AI are computed with yields denominated in term to maturity. Second, dividing the Ws in Table II by one (for unit Duration) makes the resulting ratios in Table AII comparable to the regression coefficients, since they denote the volatility of yields at a given term relative to those with unit term. Each row of Table AII estimates the regression coefficient of each Duration on unit Duration *for a given K*. But since K changes over time, there is no stable W for the regression coefficients to estimate except, in a very rough way, the one representing the average K over the regression period. Third, the actual data, published by the Federal Reserve Board, come from less than perfectly fitted yield curves. The regression coefficients obtained with these data inherit some of their errors.

Notwithstanding these problems, the regression coefficients in Table AI, roughly transformed to correspond with units of Duration, are remarkably similar to the relative Ws in Table AII. (The Durations were computed at the average yield of each term over the regression period.) The comparison is shown in Table AIII. Clearly, the orders of magnitude of the Ws and the regression coefficients are the same, a remarkable thing in itself in view of the statistical problems described above.

This result in no way depends on the arbitrarily chosen regression period, since the coefficients are remarkably stable over various fit periods. Table AIV compares the regression coefficients for periods of two, five and 10 years. In part, but only in part, this stability might reflect constraints put on the curves used to obtain the constant maturity data. But we believe it mainly reflects the stability of the underlying relationship.

By Prof R. M. Gidlow

Review of the gold sales policy of the United States Treasury (1978–79)

Roughly two-thirds of all gold holdings of central banks who are members of the International Monetary Fund (IMF) are held by seven countries – namely the United States, West Germany, France, Italy, Switzerland, Netherlands and Belgium. In recent years only one of these countries – the United States – has pursued a policy of selling gold on the free market. At the end of 1974 the American Treasury offered two million ounces of gold for sale, and in the following June sold 500 000 ounces. These intermittent sales were followed by more regular sales which started in 1978. In April of that year the United States Treasury announced its intention of holding six monthly auctions of gold, each amounting to 300 000 ounces, with the first sale to be held in May. In August 1978 the Treasury announced that as from November 1978 it would offer 750 000 ounces a month for sale. Thereafter, in November, it declared that, starting in December 1978, at least 1,5 million ounces a month would be sold. Subsequently, in April 1979, the Treasury offerings would be reduced to 750 000 ounces. In October it announced that regular monthly sales would be discontinued, although the right was reserved to sell gold in amounts and at times of its own choosing. The purpose of this paper is to review this sales policy during the period since May 1978, assess its impact, and outline future prospects for American sales of gold.

1 MOTIVATIONS BEHIND AUCTIONS

One criterion which the United States Treasury could have used to determine its sales is the relationship between American liabilities to foreign central banks and the price of gold. On this basis the United States should only sell gold when it is overvalued relative to American debt to foreign central banks. Such a guideline, however, was not followed, and instead gold sales were carried out alongside a deteriorating position as regards external indebtedness as shown in the following table:

Table 1
Gold reserves and foreign official liabilities of United States ($ millions)

	Gold reserves*	Liabilities to foreign official holders
End of 1977	44 550	131 097
End of 1978	60 750	162 567
End of November 1979	102 050	141 394

*Gold valued at free market price at end of period.
Source: Federal Reserve Bulletin, January 1980, p. A58, and gold price statistics.

If the Treasury had considered selling gold only when its gold stocks more than covered its debts to foreign central banks it is possible that such sales would have proved unnecessary. In such circumstances the dollar might strengthen on foreign exchange markets partly because of expectations that a fully fledged convertibility of the dollar into gold could be restored.

Defence of the dollar

The desirability of protecting the dollar was an important factor which influenced the sales policy. In line with the deteriorating external position the value of the dollar had been declining in foreign exchange markets since around the middle of 1977. By boosting exports of gold from the United States and/or reducing imports, gold sales helped to reduce the trade deficit which reached $26,5 billion in 1977 and $28,5 billion in 1978 following a deficit of $5,9 billion in 1976. This in turn could benefit the dollar, and therefore the incentive for investors to sell dollars could subside. At the same time this could improve the political position of America in its relations with foreign governments such as Middle East oil exporting states since countries like Saudi Arabia are large holders of dollars.

Nevertheless, only limited results could have been expected in this respect. The impact on the American trade position was negligible, particularly when the sales commenced with offerings of 300 000 ounces a month.[1]

In addition, selling gold for dollars does not allow the Treasury to accumulate foreign currencies, which is what would be desirable from the viewpoint of enhancing the ability of the American monetary authorities to intervene in foreign exchange markets to support the dollar. As the following table indicates such support is rendered more difficult by the limited foreign exchange holdings of the American authorities:

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Table of Values	1971	1974	1980
	R	R	R
Diamonds	6 000	16 491	164 912
Proof Krugerrands	6 000	n/a	131 328
Krugerrands	6 000	26 400	89 760
Stamps	6 000	9 600	80 000
Silver bullion	6 000	24 770	77 170
Rare Coins	6 000	37 500	70 000
Fixed Interest 10% per annum compounded	6 000	7 986	14 147

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Table 2
American reserve assets ($ millions)

	End of 1976	1977	1978	1979
Gold stocks*	11598	11719	11671	11172
SDR's	2395	2629	1558	2724
Reserve position in IMF	4434	4946	1047	1253
Foreign currencies	320	18	4374	3788
Total	18747	19312	18650	18937

*Gold valued at old official price of $42 an ounce.
Source: Federal Reserve Bulletin, January 1980, Washington, p. A55.

Reduction in monetary role of gold

Statements issued by the Treasury indicated that another objective of the sales was to reduce the monetary role of gold, an aim which could be helped if the sales induced a significant fall in the free market price of gold since this would reduce the value of gold stocks held by central banks. The Treasury presumably also hoped that its sales of gold would encourage other countries with large gold holdings to do likewise.

There is, however, no evidence that this has materialised. Although the volume of gold in aggregate held in the monetary reserves of the countries which report to the IMF have declined gradually since 1945 this proves nothing. Countries can easily conceal gold purchases, and in any case over this period the value of the gold holdings of central banks have increased enormously. Selling gold, moreover, in order to encourage other countries to do likewise, is inconsistent with the objective of moving towards a system in which the SDR is the principal reserve asset. The selling of gold for dollars or other currencies by central banks only increases the role which national currencies perform in the international monetary system. It can similarly be argued that selling gold even on the free market to improve the American balance of payments position involves activating the gold stocks of the Treasury as a monetary asset, which is inconsistent with the attempt to diminish the monetary role of gold.

It cannot be excluded that this attempt to reduce the role of gold in the system through gold auctions had been abandoned by the Americans by November 1978 when the monthly sales were increased to 1,5 million ounces. By that time evidence was available which suggested that the weakness of the dollar had created a desire on the part of central banks, particularly those in developing countries, to diversify their holdings of foreign assets.(2)

In late 1978 a secret accord could have been reached under which the Americans agreed to purchases of gold at its auctions by central banks as a quid pro quo for the foreign currency swaps which were provided to them by the countries with hard currencies, namely Japan, Switzerland and West Germany.(3)

Other influences

There are reasons for believing that other factors can also influence the gold sales policy of the American Treasury. A Treasury official claimed that it continually reviews its sales policy, and takes into account considerations such as the state of the American and foreign economies, inflation and oil prices.(4) This, however, does not provide an accurate guide to future sales tactics. The rising rate of inflation in the United States, for instance, together with the escalation of world oil prices in 1979, suggests that the American authorities could have increased their gold sales rather than cut them as happened. On the other hand, suggestions by the United States that it might establish a military task force for use overseas in the event of a further serious curtailment of oil supplies raised the possibility that the funds of certain OPEC members which are invested in America could be blocked. This could have made such countries more wary of holding dollars. Consequently, if the United States viewed the future supply of oil as so uncertain this provided grounds for cutting gold sales in order to render it more difficult for OPEC countries to diversify out of the dollar.

2 IMPACT ON PRICE AND STATUS OF GOLD

It is widely assumed that the Americans have been trying, inter alia, to depress the gold price through the gold auctions they conducted in 1978 and 1979. Such an outcome could support the status of the dollar as a reserve asset while diminishing that of gold. Apart from this a continued rise in the price of gold could perpetuate and intensify inflationary fears around the world and render inflation even more difficult to control.

On the other hand, it is questionable whether the United States is keen to simply depress the price of gold in view of its large holdings of more than 250 million ounces. It cannot completely ignore the domestic political implications of such action in the face of the growing popularity of gold as an investment medium in the country. A rising price of gold, moreover, does not herald a calamity where inflation is concerned as it helps to focus attention on the need for sound monetary policies which, if implemented, should eventually create greater economic stability. Nevertheless, even if this is true, there are circumstances in which the American Treasury could favour a fall in the gold price. In particular, if it believes that strength in the gold price is helping to weaken the dollar on foreign exchange markets it could welcome a decline in the gold price.

Impact on price

In so far as the sales programme was designed to weaken the gold price the results were disappointing. On the occasions when changes in sales policy were announced in April, August and November 1978 the initial effect was to weaken the price, but each time the reaction was short-lived. The Americans conducted their sales in a rising market which was not conducive to attempts to depress

the price for more than a temporary period. These sales decisions, inter alia, could have imbued some investors with confidence who had previously feared that larger tonnages would be offered for sale.[5]

Even in early November 1978, when the market was vulnerable, the announcement of a doubling of gold sales only had a temporary effect. By that time the gold price had risen sharply to a record level of $244 an ounce, and the dollar had been falling steadily, thus presenting the prospect of at least a rally in the American currency.[6] Yet the downward pressures on the gold price lasted only until early December when it reached $194 an ounce. By February 1979 the price had reached a new record of $254 an ounce.

The quick recovery of the gold price on this occasion was the result of several influences. It was reported that the Russians had stopped selling gold on the free market around the end of October 1978 after having sold roughly 400 tons during the year. This action by the Russians offset to a considerable extent the gold sales by the Americans, which amounted to roughly 47 tons a month from December 1978 until April 1979. In addition, the political situation in Iran deteriorated in November and December, and this was accompanied by disruptions in the export of oil from that country. This contributed towards the phased-in increases in oil prices of 14,5 per cent for 1979 which were announced by the members of OPEC in December 1978.

The failure of the auction programme to depress the price, except on a temporary basis, *can be partly ascribed to the political weakness of the American Administration.* The failure of the United States to support the regime of the Shah when it came under pressure in 1978 contributed to its collapse. This was associated with a new oil crisis in 1979 which led to sharp increases in petroleum prices. The accession of the anti-American clerical rulers in Iran was followed by the hostage drama in Teheran in November 1979, and the freezing of official Iranian assets in America as well as those in American bank branches abroad. All these events had a positive influence on gold, and made it much more difficult for the United States Treasury to downgrade gold through its auctions.

It is possible that the Treasury recognised that under the circumstances prevailing, its programme of auctions would not depress the price of gold for long. Nevertheless, it could have concluded that temporary sharp drops in the price, such as occurred in November 1978, would illustrate the volatility of the price of the metal, and discredit it as a monetary asset. The sharp fall in the dollar, however, had discredited the argument that gold was unstable and currencies stable.

Impact on status of gold

The failure of the auctions to check the rising price of gold for more than a short time was related to the impact of the sales on the demand for and status of the metal. In this respect the pattern of demand at the Treasury auctions as well as those of the IMF is interesting and presented in Tables 2 and 3:

Table 3
Bids, offerings and prices at IMF auctions

Auction date	Bids*	Offerings	Ratio of bids to offerings	Average price ($)
	million ounces			
3. 5.78	3,10	,525	5,91	170,40
7. 6.78	1,07	,470	2,28	183,09
5. 7.78	,79	,470	1,70	184,14
2. 8.78	1,47	,470	3,12	203,28
6. 9.78	,77	,470	1,65	212,50
4.10.78	,80	,470	1,71	223,68
1.11.78	,69	,470	1,47	224,02
6.12.78	1,96	,470	4,18	196,06
3. 1.79	1,48	,470	3,15	219,34
7. 2.79	1,49	,470	3,17	252,53
7. 3.79	1,53	,470	3,26	241,68
4. 4.79	1,19	,470	2,53	239,21
2. 5.79	1,51	,470	3,21	246,18
6. 6.79	1,45	,440	3,30	280,39
3. 7.79	1,51	,440	3,45	281,52
1. 8.79	1,14	,440	2,59	289,59
5. 9.79	1,65	,440	3,75	333,24
10.10.79	,66	,440	1,51	412,78
7.11.79	1,78	,440	4,05	393,55

*At the auctions between June 1978 and May 1979 non-competitive bids by certain central banks were also allowed.
Source: IMF Surveys, Washington.

Table 4
Bids, offerings and prices at American auctions

Auction date	Bids	Offerings	Ratio of bids to offerings	Average price ($) Good delivery	Coin
23. 5.78	1,36	,300	4,55	180,38	
20. 6.78	1,00	,300	3,34	186,91	
18. 7.78	1,38	,300	4,60	185,16	
15. 8.78	,56	,300	1,88	213,53	
19. 9.78	,77	,300	2,57	212,85	
17.10.78	,83	,300	2,78	228,44	
21.11.78	,92	,750	1,23	199,05	
19.12.78	2,71	1,500	1,81	214,17	
16. 1.79	8,01	1,500	5,34	219,77	218,22
22. 2.79	3,06	1,500	2,04	252,38	251,42
20. 3.79	3,68	1,500	2,45	241,30	240,09
17. 4.79	3,36	1,500	2,24	230,37	230,17
15. 5.79	2,40	,750	3,20		254,92
19. 6.79	2,00	,750	2,67		279,02
17. 7.79	2,10	,750	2,80		296,44
21. 8.79	2,26	,750	3,01		301,08
18. 9.79	2,60	,750	3,47		377,78
16.10.79	1,20	,750	1,60		391,98
1.11.79	1,50	1,250	1,20		372,30

Source: Finance Week, Johannesburg, November 15–21, 1979, p. 403.

The tables show that the larger monthly sales which were conducted by the United States Treasury were accompanied by larger bids than at the smaller auctions conducted by the IMF. There are a number of reasons for suggesting that the increased auctions by the Americans which started in December 1978 created a larger demand for gold. Firstly, buyers who were interested in acquiring larger tonnages than were available at the IMF auctions or in the free market, but who did not want to affect the price, were given a better opportunity once the Americans started to offer 1,5 million ounces for sale each month.[7] This opportunity existed for official buyers as well as private investors. Although the Treasury indicated that it would not knowingly sell gold to a central bank there was nothing to stop such an institution using a third party if it was keen to acquire gold at any of the auctions. Secondly, the regular supplies of gold from the Treasury made the supply variable more predictable, even though these regular sales could have created uncertainty if fears arose that the monthly auctions would be adjusted from time to time. Investors were encouraged to buy on the assumption that the Treasury would abandon or reduce sales at some stage. Thirdly, the auctions could have stimulated speculation on the future markets in the United States by creating a new source of supply to settle gold delivery commitments under future contracts.

On this basis the size of the bidding should have declined in line with the fall in the offerings from May 1979 onwards. In this respect, although demand did fall as shown in the previous table, this is misleading. The ratio of bids to the offerings increased at the May 1979 auction compared with the three previous ones. In addition, from May onwards the gold on offer was only ,90 fineness whereas at the previous auctions higher grade gold had also been sold for which a stronger demand materialised.

Apart from evidence that the larger auctions boosted demand, on occasion the results of an auction subsequently created a bigger demand for gold in the free market. The auction in August 1979 provided an example of the auction result leading the market, and causing the price to rise. At this auction the Dresdner Bank bought 720 000 ounces out of 750 000 ounces on offer. This bank is reputed to have close connections with buyers in the Middle East, and this successful large bidding led to speculation that an oil-producing state or states in the Middle East were selling dollars for gold. There is no certainty that such big buyers would operate through one bank at an auction, since they could easily place bids at varying levels through several banks. Nevertheless, this auction result encouraged gold-buying in the free market, and within two weeks the price had risen by about $20 an ounce to $320 an ounce. In other words, *this auction result stimulated the demand for gold rather than dampened it, and threatened possibly to weaken the dollar in foreign exchange markets.*

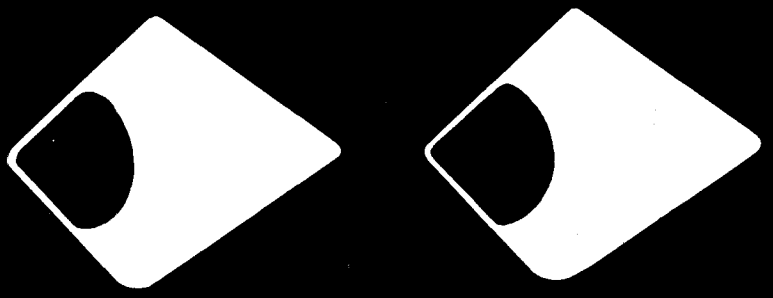
In sum, the position of gold was strengthened by the evidence provided by the auctions that the market can absorb large tonnages of gold. Future threats of large gold sales by the United States are likely to exert less impact on the market unless the environment surrounding goldnomics is different. In retrospect, the Americans could have retained greater influence over the bullion markets by staying out or by selling less, because of the greater uncertainty which they could have exerted over the market.

3 SCALING DOWN OF SALES

The policy of progressively increasing the size of the monthly auctions was reversed starting in April 1979, when it was announced that future monthly offerings at the auctions were to be reduced to 750 000 ounces.

Motivation

This move was justified by the American authorities on the grounds that the position of the dollar in the foreign exchange markets had strengthened, while gold no longer appeared to be a destabilising factor in these markets.[8] On the other hand, although the dollar had improved its position it was still suspect in view of the American



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balance of payments deficit on current account, and the high inflation rate.[9] Moreover, it is debatable whether gold, even at the time of the announcements of increased auctions in 1978, was a destabilising force. Foreign exchange markets are influenced by many different factors from day to day, and assessing the influence of gold price movements is difficult. When the gold price is rising strongly it can simultaneously be erratic, and it is questionable whether this encourages large selling of dollars to buy gold. The rising price of gold could have largely reflected the instability in the foreign exchange markets. At the same time it has been alleged that the volatility of the gold price is, to some extent, the product of American sales of gold together with the attempts by the Administration to discourage gold buying by central banks.[10] During submissions to the United States Senate Banking Committee in December 1979 the Treasury stated that it was not the purpose of its gold sales to stabilise the gold market.[11]

There are grounds for believing that other factors also played a part in the decision to reduce gold sales. Firstly, members of the American Congress had expressed unease about the sales because they represented the swapping of a real appreciating asset for depreciating paper money. The proposal of the Carter Administration in March 1979 to sell only 15 million ounces of silver from its stockpile, instead of a much higher amount which had been anticipated, could have been prompted by fears that the American Congress would oppose larger sales in view of the gold sales programme. The Treasury is also committed to start selling one million ounces of gold in the form of one ounce and half-ounce gold medallions in June 1980. The sale of medallions was forced on the Treasury by Congress as the price for conducting its monthly auctions of gold.

Secondly, the American Treasury could have concluded that by selling gold it was simply distributing it to foreign buyers who were keener to hold gold than dollars. Although it can be argued that such sales of gold did not threaten the dominant role of the dollar in the international monetary system, they were not enhancing its role. To the extent that the Treasury wishes to frustrate attempts by private and official holders of dollars to reduce the dollar component of their assets, the selling of gold is not necessarily compatible with this objective.

Thirdly, the move could have been partly influenced by manoeuvres initiated by the Europeans which related to gold. One aspect of the European Monetary System, which became operational early in 1979, is the creation of European Currency Units. These are issued to the central banks of the EEC in return for the transfer to the European Monetary Cooperation Fund of 20 per cent of each of the gold and dollar reserves of these banks. It is envisaged that gold could fulfil both a liquidity and a transaction role in the new system.[12] By promoting the monetary role of gold it makes the American pontifications that its own gold sales are furthering the cause of gold demonetisation look incongruous. The Americans could conclude that because of the European Monetary System it could at some stage become useful to settle debts in gold with members of the EEC.

Fourthly, the cut in sales could have been designed in part to relieve the upward pressure on the dollar. The decline of the major European currencies and the Japanese yen in foreign exchange markets in the period from November 1978 to April 1979, together with the rise in oil prices, threatened to contribute towards a significant deterioration in inflation rates in these countries. The reduced size of the auctions could have reflected, therefore, a desire to promote stability in foreign exchange markets.[13] The record of the auction programme, however, does not suggest that varying the amounts of gold on offer would have any meaningful effect upon the position of the dollar in the markets.

Abandonment of regular sales

On 16 October 1979 the Treasury announced that regular monthly gold sales were to be discontinued, and in future auctions would be held at the discretion of the Treasury and in flexible amounts. The main motivation behind the change was probably to discourage the buying of gold by short-term operators, which helped to push the price up and in turn possibly weaken the dollar in foreign exchange markets. Buyers of gold under the new circumstances could never be sure what the Treasury was going to do. In particular, the new policy could have induced some investors to reduce their short-term gold holdings, because of the greater danger of price fluctuations that could follow a sudden statement by the Treasury concerning sales of gold.

These conclusions will not be valid, however, if the new policy turns out to be one, largely if not completely, of refusing to sell gold. Some commentators viewed the October decision as a face-saving formula which in practice meant that the Treasury was abandoning its sales of gold. Already, the decision to cut sales in April 1979 had demonstrated the reluctance of the American Treasury to sell its good delivery gold, since from then onwards until October 1979 it sold only gold of ,90 fineness.[14] If the dollar had been strong in October 1979 and the gold price weak an unequivocal statement on the abandonment of sales could have been made. Unfortunately, this was not the case. The balance of payments position militated against such a move, as well as the danger that it would lead to a sharp increase in the gold price, and thereby possibly weaken the dollar. In contrast, the decision to cut sales in April 1979 occurred after the gold price had fallen to around $231 an ounce from a record $254 an ounce in February 1979.

The claim that the move represented a phasing out of sales was cast in doubt when the Treasury quickly announced that it would sell up to 1,25 million ounces of gold on 1 November 1979. In the subsequent three months, however, no sale was made despite a further rise in the gold price from around $375 an ounce at the beginning of November to $850 an ounce in late January 1980. The reason for this was undoubtedly the repercussions of the seizure of the American Embassy staff in Teheran by militant Iranian students in November 1979, and in particular the freezing of official Iranian assets in American banks and their branches abroad by the Carter Administration. This action threatened to accelerate the trend towards diversification out of the dollar, and consequently any gold auctions would have facilitated this process. In other words, *the freezing of the Iranian assets undermined to some extent the rationale behind the gold auction programme.*

Impact on dollar

The abandonment of regular gold auctions in October 1979 could be interpreted as a relaxation of American efforts to strengthen the dollar. The monthly sales of 750 000 ounces, if continued, would have reduced the trade deficit of the country by $3 600 million on an annual basis assuming an average gold price of $400 an ounce. In view of the extra burden which was placed on the United

States current account position by the higher oil prices imposed by OPEC at the end of 1979, any scaling down of sales could have diminished the prospects for a strengthening of the dollar. Even so, although the absolute impact of the oil price increases was the greatest for the United States because it is the largest importer of oil, the American economy is relatively less dependant on imported oil compared with most other economies.

The results of the auctions, moreover, strongly suggested that the position of the dollar is much more influenced by fundamental factors such as American monetary policy rather than the amount of gold which is sold.

4 ALTERNATIVE SALES POLICIES

During the 1978-79 period proposals were put forward from time to time recommending changes in the gold sales policy of the American Treasury. In particular, the Treasury was urged on occasion to undertake more substantial monthly sales in an effort, inter alia, to boost the dollar, reduce the trade deficit, and depress the price of gold.[15] Such a strategy, however, would have been subject to a number of drawbacks. Firstly, more substantial sales could have attracted even greater buying by investors who were interested in acquiring large tonnages. Secondly, increasing the size of the auctions could have resulted in a more significant de facto restoration of dollar convertibility into gold at the official level. By the second half of 1979 various reports indicated that certain central banks were buying gold.[16] Thirdly, the American authorities appreciate the esteem with which people throughout the world regard gold. The need to maintain a war chest limits the extent to which gold can be sold. In the face of a rate of inflation of around 13 per cent which prevailed in the United States in 1979, it is questionable to what extent the dollar could be relied upon in emergency situations. Fourthly, dwindling United States gold reserves could cause a further erosion of confidence in the dollar, although this is open to doubt if domestic monetary policies were fostering monetary stability. Finally, the presence of the IMF gold auctions was a constraint on larger American sales since any marked decline in the gold price would have adversely affected the size of the IMF Trust Fund which was designed to provide cheap loans to poor developing countries.[17]

On the other hand, there has been a body of opinion, even in the United States, which consistently opposed the sale of gold by the Americans in the 1978-79 period. One conclusion of this paper has been that these gold auctions had a negative impact from the viewpoint of the United States. At the same time there are clear signs that gold is continuing to benefit from a refurbished role as a monetary asset in the international monetary system. One indication of this is the role of gold in the new European Monetary System. These factors suggest that a change in attitude and policy towards gold by the American Treasury could materialise in the not too distant future.

References

1 At a press conference in early November 1978 the Unitd States Treasury Secretary Mr M. Blumenthal said that on an annual basis monthly sales of 1,5 million ounces of gold would improve the American balance of trade by $4 billion. See *IMF Survey,* Washington, Volume 7, No. 21, 6.11.78, p. 349.

2 For a review of attitudes of central banks towards the question of diversifying away from the dollar see *Euromoney,* London, October 1978, pp. 31-43.

3 As part of the measures announced on 1 November 1978 increases were arranged in the United States Federal Reserve Bank's reciprocal currency swaps with the central banks of West Germany, Japan and Switzerland by $7,6 billion to $15 billion. See *IMF Survey,* op. cit., p. 348.

4 See statement by Fred Bergsten, Assistant Secretary of the Treasury for International Affairs in *Green's Commodity Market Comments,* New York, Vol. XIV, No. 6, 28.3.79, p. 2.

5 Henry Jarecki, 'Why United States Treasury Sales Are Not Bringing Down the Price', *Euromoney,* London, August 1978, p. 89.

6 From the beginning of 1978 to the end of October 1978 the American dollar had declined by roughly 35 per cent against the Japanese yen, 30 per cent against the Swiss franc, and 20 per cent against the Deutschemark.

7 From December 1978 to April 1979 around 4,4 million ounces of gold were sold each month by gold producers and through auctions. South Africa sold roughly 1,9 million ounces, other producers about 500 000 ounces, the United States 1,5 million ounces, and the IMF 470 000 ounces.

8 *IMF Survey,* Washington, Volume 8, No. 8, 23.4.79, p. 132.

From the time of the measures which were announced to support the dollar at the beginning of November 1978 until the end of April 1979 the dollar performed well. It rose by roughly 25,5 per cent against the Japanese yen, 17 per cent against the Swiss franc, and 10,5 per cent against the Deutschemark.

9 In 1978 the deficit on current account was $13 478 million while most estimates for 1979 suggest it was in the range of $10 to $12 billion. Meanwhile, consumer prices rose by 13 per cent on an annual basis in the first four months of 1979.

10 Guillaume Guindey, *The International Money Tangle,* Blackwell, Oxford, 1977, pp. 64-5.

11 *International Herald Tribune,* New York, 5 and 6 January 1980, p. 9.

12 For further details see R. M. Gidlow, 'The European Monetary System and Its Implications for Gold', *The South African Banker,* Vol. 76, No. 3, August 1979, pp. 152-4.

13 Speaking in Washington on 15 May 1979 the American Federal Reserve Board Chairman William G. Miller said that the dollar support programme of November 1978 had been highly successful in stabilising the dollar. Nevertheless, he thought that during the first few months of 1979 the yen had fallen too far, and the United States did not want to see instability return to the foreign exchange markets.

14 Around the end of 1978 fine gold of ,995 fineness (i.e. good delivery gold) accountd for roughly 18 per cent of the United States gold stock of roughly 270 million ounces. About 73 per cent of the gold stock was coin gold bars of ,90 fineness, and this has to be refined to take out the impurities before it becomes good delivery gold. The balance was imparted gold.

The January to April 1979 auctions by the Treasury consisted of one million ounces of good delivery gold and 500 000 ounces of the lower quality gold. If the Treasury in future decided to undertake larger sales of gold of ,90 fineness it could face difficulties because of insufficient capacity of gold refiners to handle the tonnage involved. It could necessitate a longer waiting period before buyers took delivery and an increased discount on the free market price.

15 For an analysis of the case for massive gold sales by the United States and the IMF together as an anti-inflationary device see M. Bronfenbremer, 'On Dumping Gold', *The South African Journal of Economics,* December 1978, pp. 352-59.

16 According to one report Saudi Arabia bought 60 tons of gold from Russia in 1979 which cost around $600 million. See *Sunday Telegraph,* London, 9.9.79, p. 19. Another report indicated that the Turkish central bank had been buying gold. See *Sunday Telegraph,* London, 6.10.79, p. 19. On 25 October 1979 the deputy bullion manager of the Swiss Bank Corporation, speaking in London, said a number of small central banks which had little gold in their reserves had been buying gold even at prices approaching $400 an ounce. In a speech in Chicago on 19 September 1979 the Chairman of Credit Suisse, Rainer Gurt, claimed that central banks in the Middle East were buying gold. The Italian newspaper La Republica, reporting an interview with the former Bank of Italy director general Rinaldo Ossola on 10 October 1979, echoed similar views.

17 In May 1980 these gold auctions were discontinued by the IMF. This does not necessarily mean that this constraint on US gold sales has been eradicated, since the IMF indicated at the September 1980 meeting of the Fund that it may raise loans from the international capital markets. This raises the possibility that it may use part of its gold holdings as collateral for such loans. In these circumstances the IMF cannot be completely indifferent to the trend in the gold price.

By S. T. Goddard, S. A. Michener and J. A. Rickard*

An illustrative method for valuing annuities

*Graduate School of Business Administration and Department of Mathematics, University of Melbourne

1 INTRODUCTION

In many situations, individuals or institutions wish to know an equivalent dated value of a series of cash flows. If the cash flows are of equal value and occur at equal intervals of time, they are said to take the form of an *annuity*.

The *future value* of an annuity is the sum of all the cash flows and the compound interest on them accumulated from the beginning of the first payment interval to the end of the last payment interval of the annuity.

In many situations, a cash flow occurs at the termination of an annuity which is not part of the series of regular payments. Such a cash flow is known as a *balloon payment* and must be considered when finding the present value of the annuity.

The *present value* of an annuity is the sum of the present values of all the cash flows of an annuity and the present value of the balloon payment.

Although the values of an annuity can be readily determined by tables and modern calculators, the specific answer obtained provides no measure of the sensitivity of the result to changes in the variables. In Section 4 a graphical method for calculating the value of an annuity is presented in the form of a nomogram. The nomogram that has been constructed is easy to use (in fact the user only requires a ruler and pencil) and clearly illustrates the changes in the value of the annuity when it is subject to changes in the values of the input parameters. An example of the benefits of using the nomogram for this type of sensitivity analysis can be seen in Section 5.

2 TYPES OF ANNUITIES

In this paper, expressions will be derived for the valuation of the four main types of annuities that occur in practice.

An *ordinary annuity* arises when the cash flows occur at the end of the periods, such as, for example, the repayments on a loan or mortgage. If the payments are made at the beginning of the periods, the annuity is known as an *annuity due*. This type of annuity often occurs in a rent or lease agreement.

The next two types of annuities are both extensions of the above definitions and depend on whether the original annuity is an ordinary annuity or an annuity due. If the payments of an annuity do not commence until a number of compounding periods into the future, the annuity is known as a *deferred annuity*. For example, when purchasing a car, the buyer may be allowed a certain period of "free" use before repayments begin. If the series of equal cash flows continues forever, the annuity is known as a *perpetuity*. An example of a perpetuity is the dividends paid on non-redeemable shares.

3 CALCULATING THE PRESENT AND FUTURE VALUES OF ANNUITIES

In general, for simplification, an annuity is constructed so that the compounding of interest occurs on the same date as the annuity payments. The analysis presented in this section will be subject to this assumption, however the assumption will be relaxed in Section 6.

To define the variables of this problem, let

n = total number of payments
i = nominal interest rate (per compounding period)
PMT = periodic payment
BAL = balloon payment

Future values

For an ordinary annuity, the payments are made at the end of the periods, so the final payment will have a future value of PMT. The second last payment will be compounded for one period, giving a value of PMT$(1+i)$. This argument follows for all other payments, thus giving the future value of the first payment as PMT$(1+i)^{n-1}$. Hence, the future value of the ordinary annuity, FV_o, is

$$FV_o = PMT\left[1+(1+i)+\ldots+(1+i)^{n-1}\right] \quad (1)$$

Since (1) is a geometric progression with initial term PMT and common ratio $(1+i)$,

$$FV_o = PMT\left[\frac{(1+i)^n-1}{i}\right] \quad (2)$$

A similar method can be used to derive the future value of the annuity due, remembering that the payments are made at the beginning of the periods, so an "extra" period of interest applies to each term. Thus, the future value of the annuity due, FV_d, is

$$FV_d = PMT\left[(1+i)+(1+i)^2+\ldots+(1+i)^n\right] \quad (3)$$

which reduces to

$$\left.\begin{aligned} FV_d &= PMT\,(1+i)\left[\frac{(1+i)^n-1}{i}\right] \\ &= (1+i)FV_o. \end{aligned}\right\} \quad (4)$$

By definition of the future value of an annuity, the future value of a deferred annuity is calculated in the same way as the future value of an ordinary annuity or an annuity due. However, in the case of a deferred annuity, the amount of the future value is not available until the term of the deferred annuity is finished, which is longer than the term of an ordinary annuity or an annuity due by the period of deferment.

As previously defined, the future value of an annuity is the sum of all the cash flows and the compound interest accumulated on them to the end of the term of annuity. For a perpetuity, the payments continue forever thus making the future value of the perpetuity indeterminate.

Present values

To determine the present value of an annuity, the present values of both the regular payments and the balloon payment must be considered. Although the addition of the balloon payment term complicates the expressions further, the applications of the expressions are considerably extended. For example, the balloon payment term may refer to a final extra payment that is used to pay off a loan completely, or to the selling of a piece of leasing equipment

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that has earned regular payments as rent. As stated in Section 1, the balloon payment is assumed to be made at the termination of the annuity; that is, at the end of period n for each type of annuity.

Since the present value of an annuity is the sum of the present values of all the regular payments and the present value of the balloon payment, the present value of an ordinary annuity, PV_o, is

$$\begin{aligned} PV_o &= (1+i)^{-n}\left\{PMT\left[1+(1+i)+\ldots+(1+i)^{n-1}\right]\right\} \\ &\quad + BAL(1+i)^{-n} \\ &= PMT\left[\frac{1-(1+i)^{-n}}{i}\right] + BAL(1+i)^{-n}. \end{aligned} \tag{5}$$

Similarly, the present value of an annuity due, PV_d, is

$$\left.\begin{aligned} PV_d &= (1+i)^{-n}\left\{PMT\left[(1+i)+(1+i)^2+\ldots+(1+i)^n\right]\right\} \\ &\quad + BAL(1+i)^{-n} \\ &= PMT(1+i)\left[\frac{1-(1+i)^{-n}}{i}\right] + BAL(1+i)^{-n}. \\ &= (1+i)PV_o - iBAL(1+i)^{-n}. \end{aligned}\right\} \tag{6}$$

The expression for the present value of a deferred annuity can be found by assuming that there are n+m equal payments to be made with the first m payments withheld. This would then be equivalent to making n payments beginning at a time of m periods into the future.

For an ordinary annuity, the present value of the deferred ordinary annuity, PVD_o, can be found by assuming that all n+m payments were made and then subtracting the present value of the first m payments from the present value of all the payments and the present value of the balloon payment. Thus

$$\begin{aligned} PVD_o &= PMT\left[\frac{1-(1+i)^{-n-m}}{i}\right] + BAL(1+i)^{-n-m} \\ &\quad - PMT\left[\frac{1-(1+i)^{-m}}{i}\right] \\ &= PMT\left[\frac{(1+i)^{-m}\{1-(1+i)^{-n}\}}{i}\right] + BAL(1+i)^{-n-m}. \end{aligned} \tag{7}$$

A similar method can be used to find the present value of the defered annuity due, PVD_d, giving

$$\left.\begin{aligned} PVD_d &= PMT(1+i)\left[\frac{1-(1+i)^{-n-m}}{i}\right] + BAL(1+i)^{-n-m} \\ &\quad - PMT(1+i)\left[\frac{1-(1+i)^{-m}}{i}\right] \\ &= PMT\left[\frac{(1+i)^{-m+1}\{1-(1+i)^{-n}\}}{i}\right] + BAL(1+i)^{-n-m} \\ &= (1+i)PVD_o - i\,BAL(1+i)^{-n-m}. \end{aligned}\right\} \tag{8}$$

Since the payments of a perpetuity continue forever, the expression for the present value of a perpetuity may be found by letting n tend to infinity and determining the limiting value of the present value of the annuity. Thus, for the present value of an ordinary perpetuity, PVP_o, the expression is

$$\begin{aligned} PVP_o &= \lim_{n\to\infty}\left\{PMT\left[\frac{1-(1+i)^{-n}}{i}\right]\right\} \\ &= \frac{PMT}{i} \end{aligned} \tag{9}$$

since $\lim_{n\to\infty} \frac{1}{(1+i)^n} = 0$ for all non-negative i.

Similarly, the present value of a perpetuity due, PVP_d, is

$$\left.\begin{aligned} PVP_d &= \lim_{n\to\infty}\left\{PMT(1+i)\left[\frac{1-(1+i)^{-n}}{i}\right]\right\} \\ &= PMT\left(\frac{1+i}{i}\right) \\ &= (1+i)PVP_o. \end{aligned}\right\} \tag{10}$$

When considering perpetuities, no balloon payment need be taken into account since the payments continue forever, which implies that no final payment exists.

4 THE NOMOGRAM

As previously mentioned, a graphical method of calculation of an equivalent dated value of an annuity in the form of a nomogram is presented. A nomogram is a collection of graphs that share common axes. Nomograms have found recent applications in a number of areas, including the valuation of options and warrants [1] and the comparison of flat, nominal and effective rates of interest [2].

Future values

Firstly, a nomogram will be constructed so as to give future values of all the annuities. The nomogram is constructed using equations (2) and (4).

Initially, a set of contours n = constant are used to plot i against

$$\frac{(1+i)^n-1}{i}.$$

After these curves are drawn, a set of contours PMT = constant may be used to plot

$$\frac{(1+i)^n-1}{i}.$$

against FV_o. The value of FV_o may then be found from the FV_o-axis by using the suitable contour PMT = constant. Plotting FV_o against FV_d with contours i = constant then allows the value of FD_d to be found from the FV_d-axis.

Since the equations for the future values of the annuities are quite simple, the fourth quadrant of this nomogram is not required (see Figure 1).

Example (see Figure 1)

"Find the future values of the annuities with regular payments of $100 every 6 months for 15 years, if the nominal interest rate is 4% per 6 months."

Firstly read off the nominal interest rate of 4% on the i – axis

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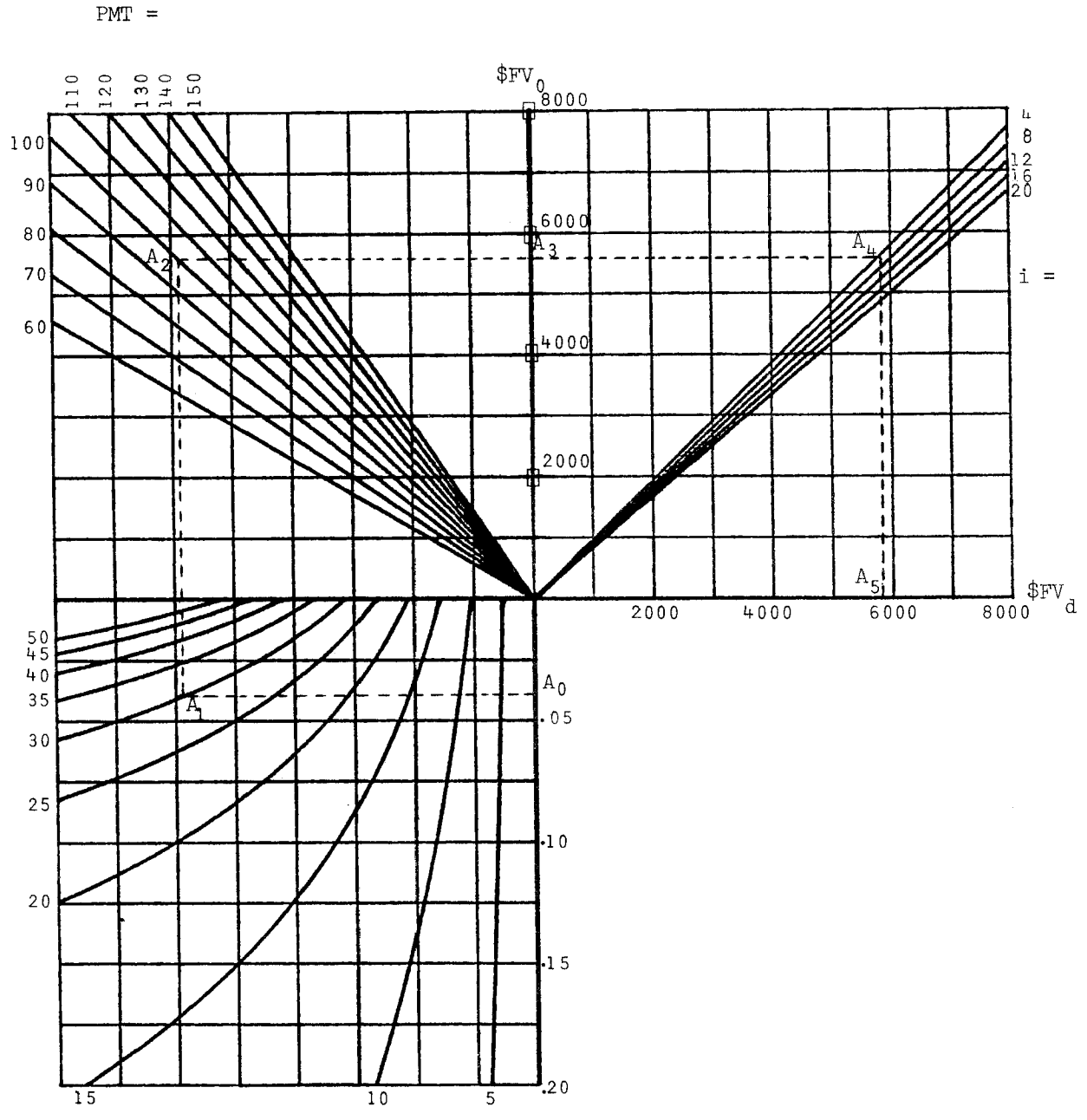


Figure 1
Nomogram for calculating the future value of ordinary annuities and annuities due.

at A_0 and draw a horizontal line to the left until it intersects the n = 30 contour at A_1. Next, draw a vertical line at A_1 until this line intersects the PMT = \$100 contour at A_2. A horizontal line is constructed at A_2 and drawn until it intersects the FV_0 – axis at A_3 which gives the value of FV_0. The last horizontal line, A_2A_3, may be continued until it meets the i = 4% contour at A_4. The vertical line constructed at A_4 intersects the FV_d – axis at A_5 which gives the value of FV_d. This solves the future value annuity problem, giving

FV_0 = \$5 610.00 and FV_d = \$5 840.00,

which yields errors of less than 1% in each case when compared to the actual values FV_0 = \$5 608.49 and FV_d = \$5 832.83.

As previously mentioned, the nomogram also gives the approximate future values of the deferred annuities

FVD_0 = \$5 610.00 and FVD_d = \$5 840.00,

but the amounts are not available until the term of the deferred annuity is finished.

Present values

As might be expected from the more complicated

formulae, the nomograms for the present values of all the annuities are more complex, and each type of annuity requires its own nomogram.

For both the ordinary annuity and the annuity due, a similar method as described above is used to construct a nomogram (see Figures 2 and 3), except that five sectors are required in each case. There is, however, one large difference in the use of these nomograms. Apart from being able to find the present value of only one type of annuity per nomogram, the value is obtained from the contours in the first sector rather than being read off an axis. Although this may cause the need for some interpolation, the number of curves in the first sector should make any errors very slight.

Example

"Find the present value of the annuities with regular payments of $100 every 6 months for 15 years if the nominal interest rate is 4% per 6 months, and the balloon payment is $500."

Firstly, consider the case of an ordinary annuity (see Figure 2). Read off the nominal interest rate of 4% on the i – axis at B_0 and draw a line perpendicular to the axis until it meets

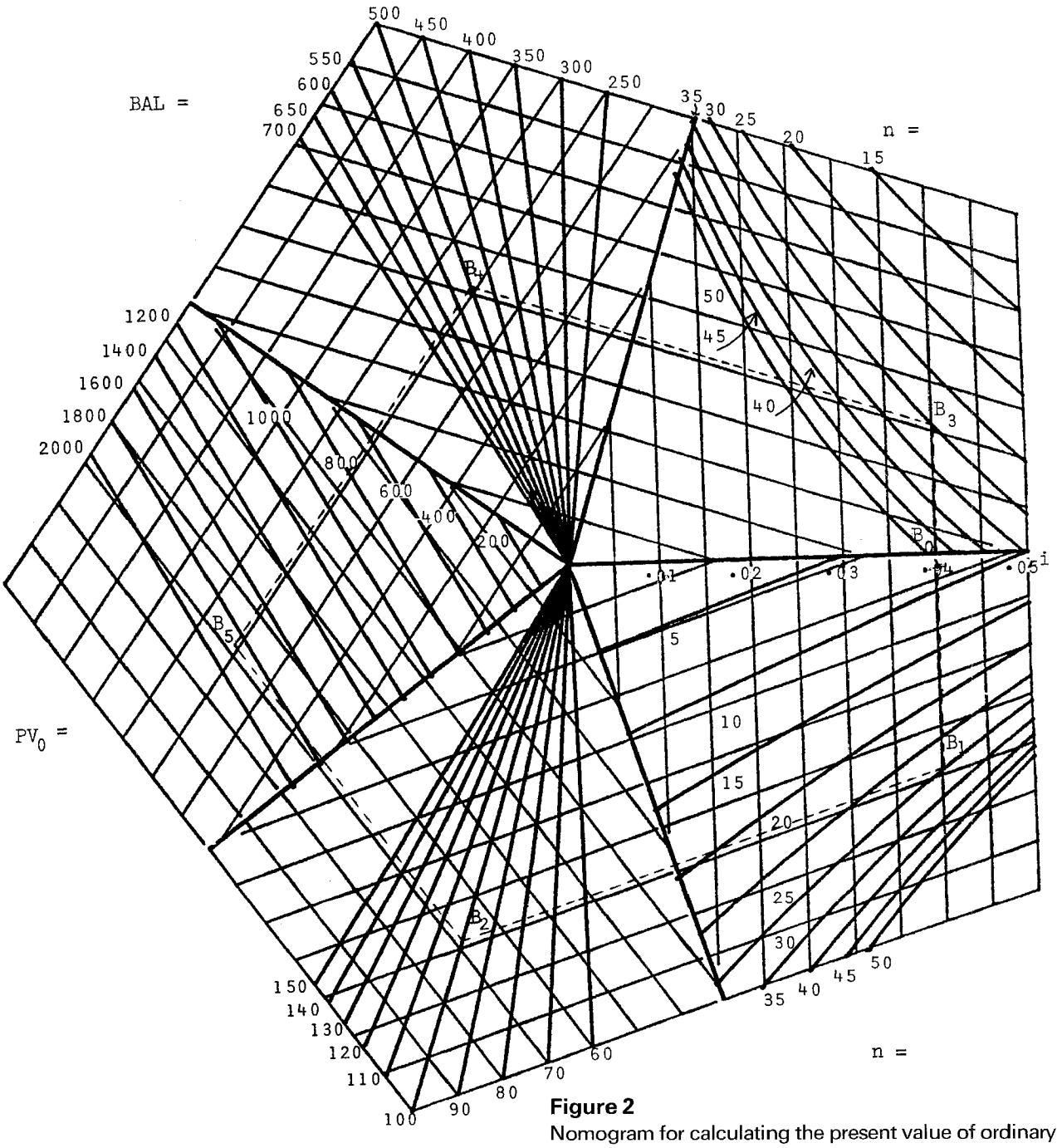


Figure 2
Nomogram for calculating the present value of ordinary annuities.

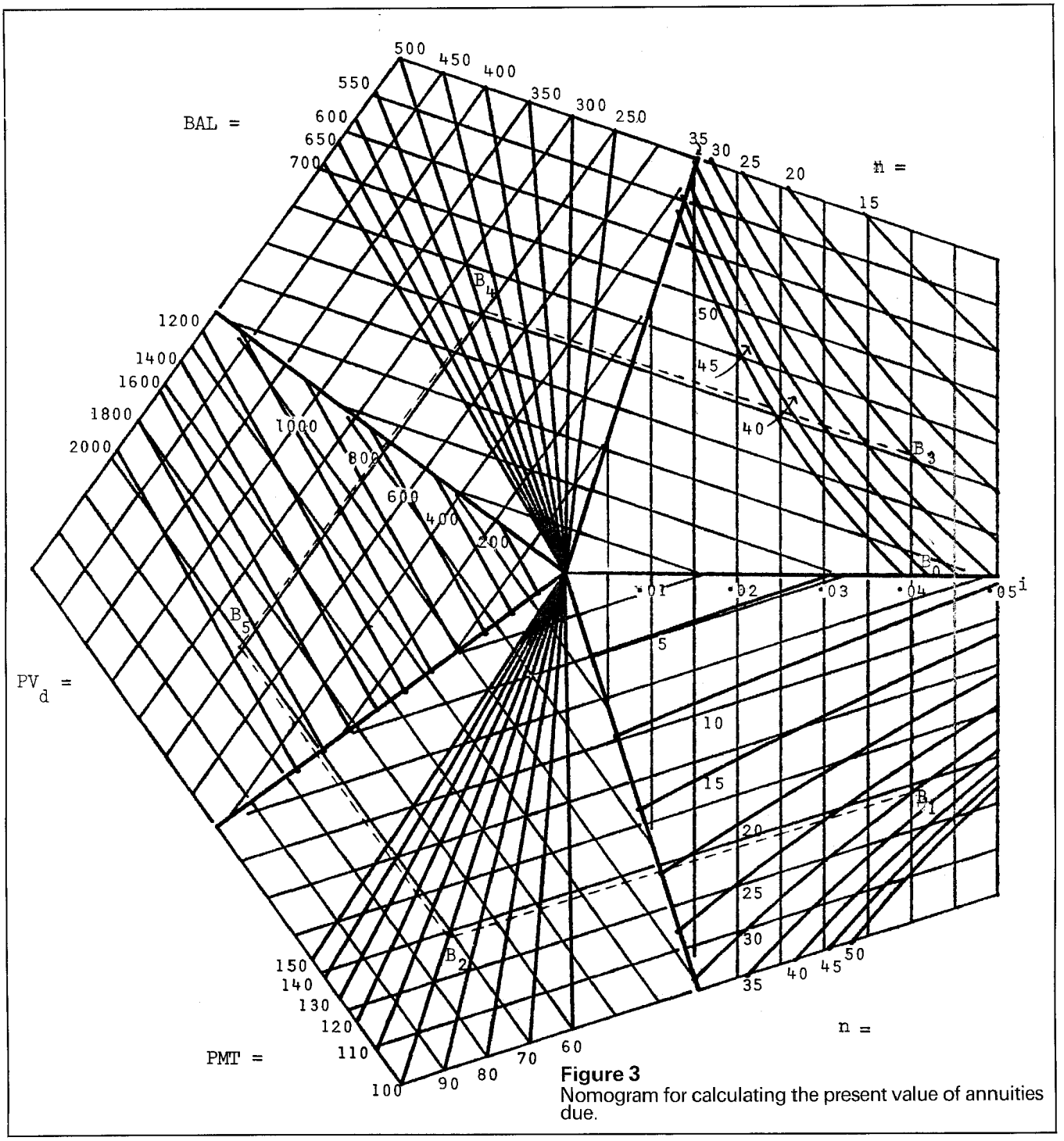


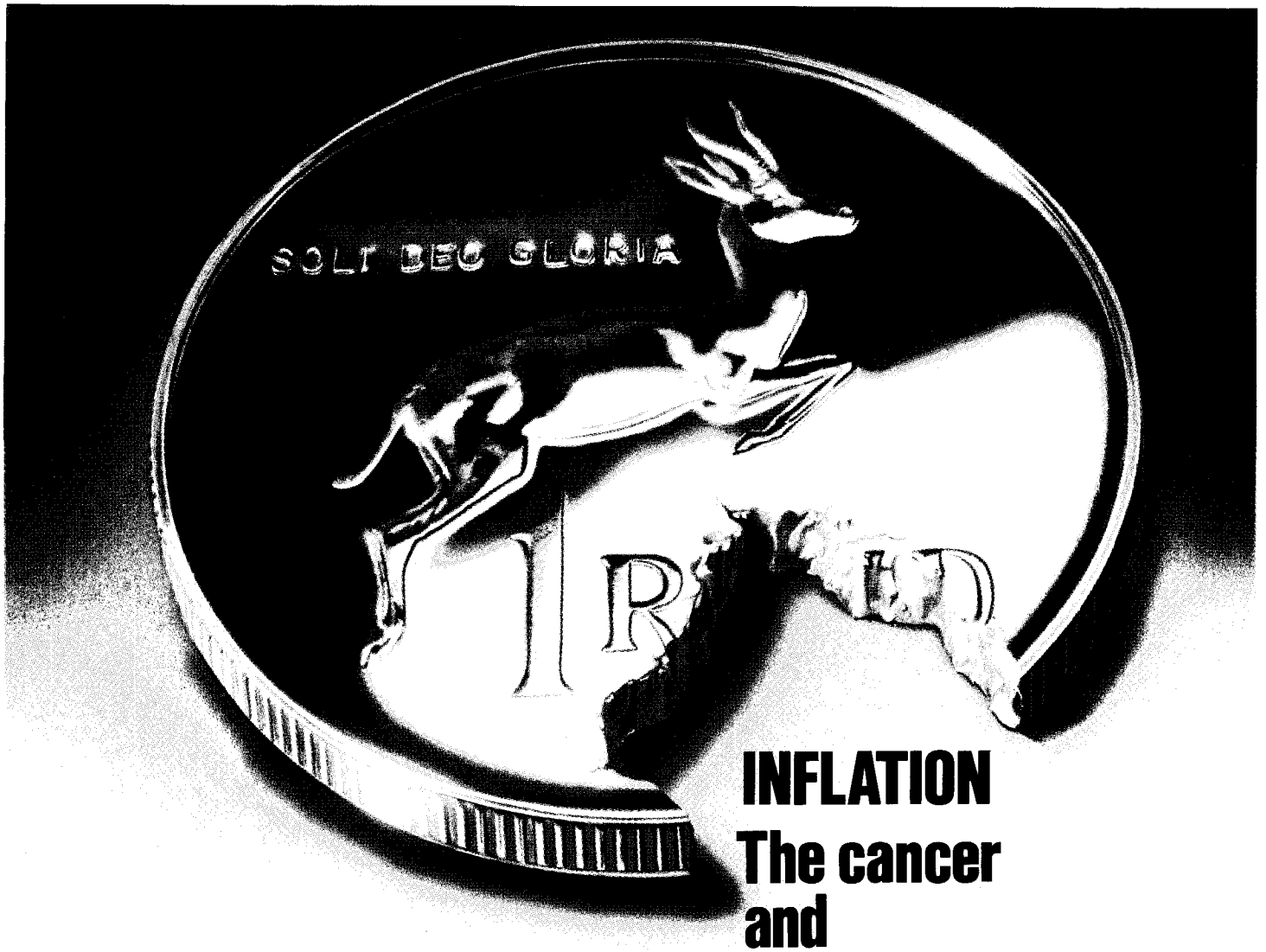
Figure 3
Nomogram for calculating the present value of annuities due.

the n = 30 contour at B_1, then draw a line parallel to the grid from B_1 to the PMT = \$100 contour at B_2. At B_2, draw a line parallel to the grid which passes through all the contours in the next sector.

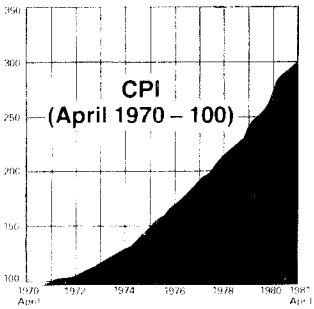
Next, returning to B_o, draw a line perpendicular to the i – axis until it meets the contour at B_3. Draw a line parallel to the grid from B_3 to the BAL = \$500 contour at B_4. Construct a line from B_4 parallel to the grid until this line intersects the line from B_2 at B_5. Using the PV_o = constant contours and the point of intersection, the value of PV_o = \$1 880 may be interpolated, which yields an error of less than 1% when compared to the correct answer of PV_o = \$1 883.36.

The solution to this example for an annuity due is shown in Figure 3 where the value PV_d = \$1 950 is obtained, yielding an error of less than 1% since the correct answer is PV_d = \$1 952.53. These two monograms thus solve the present value problem for these two types of annuities.

Although no nomograms are presented for calculating the present values of deferred annuities, the values can be found indirectly from Figures 2 and 3 by using an annuity of length n+m instead of n and then subtracting



INFLATION
The cancer and the cure



The graph illustrates the escalation in the cost of living since 1970 as measured by the S.A. Consumer Price Index.

	PRICE 1970	PRICE 1981	PRICE 1991
Litre of petrol	9,2c	61,0c	?
Small car	R2 315	R7 795	?
Loaf of bread	9,4c	42,0c	?
Carton of milk	17,0c	55,0c	?

Examples of the alarming rate at which the purchasing power of money has reduced.

Assuming	1981	1986	1991
7½ % pa	R10 000	R6 966	R4 852
10% pa	R10 000	R6 209	R3 855
12½ % pa	R10 000	R5 549	R3 079

The purchasing power of R10 000 will reduce as follows, assuming various rates of inflation.

The rate of inflation is such that even if you have made provision for the future, the chances are that the purchasing power of your savings has been painfully eaten away.

Fortunately, the Prudential has a cure. The Company was founded in 1848, and has been operating in South Africa since 1932. Over the years the Prudential has acquired vast experience and investment expertise, which is essential in combating inflation. In fact, influential publications have illustrated in surveys, how the Prudential Prubond linked funds outperformed all others over a five year period.

Yes, the Prudential has an enviable track record of successful performance! Why not do yourself a favour – contact the Man from the Prudential, who will show you how to fight inflation before it's too late.

Prudential
People helping people

The Prudential Assurance Company of South Africa Limited. Part of one of the world's great insurance groups.

$$PMT\left[\frac{1-(1+i)^{-m}}{i}\right]$$

for the ordinary deferred annuity and

$$PMT(1+i)\left[\frac{1-(1+i)^{-m}}{i}\right]$$

for the deferred annuity due. Note that the two values to be subtracted can also be found from Figures 2 and 3, using a term of annuity of length m.

The nomograms used in the above example were of normal desk pad size (45 cm x 65 cm), which explains the accuracy obtained.

5 SENSITIVITY ANALYSIS

In this section, the benefits of the nomogram for sensitivity analysis will be illustrated by changing the values of the variables of the problem. By choosing one variable and changing its value by certain amounts, the nomogram can be used to measure the sensitivity of the annuity to changes in that particular variable.

For example, Figure 2 shows the changes in the present value of an ordinary annuity when the regular payments are increased and decreased by $10, yielding present values of $2 050.00 and $1 550.00 respectively (with errors of less than 1%).

Another interesting application of sensitivity analysis is to consider the effects of changing the interest rate. In Figure 3, some valuations are shown for the present value of an annuity due where the interest rate is varied while keeping the regular payments and balloon payment constant. The present value for the above example (with i = .04) is shown, as are the present values for the example with i = .03 and i = .05, yielding present values of $2 240.00 and $1 710.00 respectively (with errors of less than 1%).

This type of analysis can be very useful when deciding the value of regular and balloon payments with respect to the changes in the present and future values of the annuity.

6 GENERAL ANNUITIES

As previously mentioned, some restrictions have been placed on the problem so far with regard to the dates of compounding of interest and the annuity payments. For the case of a general ordinary annuity, let

GPMT = number of annuity payments
s = number of compounding periods per annum
t = number of general annuity payments per annum
i = nominal interest rate per compounding period

It is possible to replace this general ordinary annuity with an equivalent ordinary annuity with payments made at the end of the compounding periods, providing that the interest rates of the two annuities are equivalent and that their values on any date are the same. Let PMT be the ordinary annuity payment which replaces GPMT and j the nominal interest rate per general annuity payment period that is equivalent to i per compounding period.

The relationship between i and j is given by

$$(i+j)^t = (1+i)^s \qquad (11)$$

which may be solved explicitly for j to give

$$j = (1+i)^{s/t} - 1. \qquad (12)$$

Since the values of the annuities are equal on any date,

$$PMT\left[\frac{(1+i)^s-1}{i}\right] = GPMT\left[\frac{(1+j)^t-1}{j}\right] \qquad (13)$$

However, from the equivalence of the interest rates in (11), (13) reduces to

$$\frac{PMT}{i} = \frac{GPMT}{j}$$

Using (12), the payment of the equivalent ordinary annuity is

$$PMT = i\,GPMT\left[(1+i)^{s/t}-1\right]^{-1} \qquad (14)$$

from which the nomograms in Figures 1 and 2 may be used to calculate the equivalent future value and present value of the general ordinary annuity.

The case of a general annuity due may be treated in an analogous manner. Assuming the variables as defined above, the general annuity due can be transformed into a general ordinary annuity by transferring each GPMT to the end of each year. This means that equation (14) could then be used, but the values of the general annuity due would exceed the value of the ordinary general annuity by

$$GPMT(1+i)^s - GPMT \qquad (15)$$

over each one year period. However,

$$GPMT(1+i)^s - GPMT = i\,GMPT\left[\frac{(1+i)^s-1}{i}\right] \qquad (16)$$

which is the future value of an ordinary annuity with regular payments i GPMT per interest period for one year. Since the general annuity payments GPMT are replaced by equivalent payments PMT made per compounding period, the payments PMT are both i GPMT more than they would be if the general annuity were an ordinary one. Thus

$$PMT = i\,GPMT\left[(1+i)^{s/t}-1\right]^{-1} + GPMT\,i$$

$$= i\,GPMT\left[1-(1+i)^{s/t}\right]^{-1} \qquad (17)$$

Again, equation (17) may be used in Figures 1 and 3 to calculate the future value and present value of the general annuity due.

It is also possible to generalise the results obtained by considering continuous compounding of interest, rather than the cases where interest is added to principal at the end of discrete periods of time. This is equivalent to interest compounded s times per year, where s tends to infinity. That is,

$$\lim_{S\to\infty}\left(1+\frac{i}{s}\right)^s = e^i. \qquad (18)$$

Thus, the effective rate of interest is

$$r = e^i - 1. \qquad (19)$$

Since this case is not in general use, it is only noted here and not considered in detail.

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[1] E. Dimson, "Instant Option Valuation", paper presented at the conference on "Investing in Options", London, November 1976.

[2] J. A. Rickard, "A nomogram relating flat, nominal and effective rates of interest", *Accountancy*, (1979), to appear.

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By A C Valsamakis

Investment basis IX

The use of comprehensive cash flow analysis as a basis for risk measurement and determination of debt capacity.

The problem of deciding whether it is prudent and proper for a business corporation to finance long-term capital needs through debt, and, if so, how much, is one which management needs to resolve at one time or another. Decisions concerning the financing mix may properly be regarded as the central problem of finance; of critical importance because of its potential impact on margins of profitability and solvency. For all companies, however large and financially sound they may be, the decision is one to be taken with great care.

Critical issues surrounding debt employment

It is beyond the scope of this article to give an appraisal of the debate that has developed about the existence of a desired level of debt viz-a-viz equity finance that gives rise to an optimal financial structure. In the light of the very 'practical nature' of firms' approaches to suppliers of long-term debt capital to finance growth/operations, such a discussion could be seen as too academic. Suffice it to say that, given the present-day tax considerations, the employment of debt levers rates of return earned on equity over the returns available on assets. This, of course, presupposes that the return on assets exceeds the cost of debt. Nevertheless, the advantage of financial leverage must not be viewed in isolation. The introduction of debt places onto already risky profits (business uncertainty) a fixed and prior charge. The need to meet fixed interest payments out of the firm's uncertain cash flows from operating may increase the variability (financial risk) of the residual earnings accruing to shareholders. In short, the problem of the debt decision is a trade-off of higher prospective income to shareholders against greater chance of loss.

Financial risk restated

When the word 'risk' is applied to debt, the precise meaning is not always clear. It may refer to a variety of potential penalties. Fundamentally, as far as debt is concerned, risk is the chance of running out of cash. It is, however, the ultimate hazard of running out of cash to the extent that firms default on legal contracts which lurks in the background of every debt decision. In such situations bankruptcy occurs and normal operations cease.

One may choose to use the term "cash inadequacy" to refer to the problems involving the inability to make cash payments for any purpose important to the long-term financial health of the business; and "cash insolvency" in the extreme case of cash inadequacy. It should be emphasized that although debt necessarily increases the chances of cash inadequacy, the risk exists whether the firm has any debt or not. The debt-equity and debt capacity decision does not revolve around the assumption of some or no risk at all; rather it concerns the degree of risk taken.

Conventional approaches to determining a firm's debt capacity and their limitations

Present-day business practice suggests that concepts of debt capacity are drawn from one or more of several sources. These may include seeking the counsel of institutional lenders, following the practices of the past, or simply refering to "common knowledge" and "general practice". Without denying the practical significance of some of these considerations, one must recognise the limitations in using them as the only guides to appropriate debt capacity.

Seeking outside counsel is tantamount to equating the risk to borrower, to that of lender. Yet, the standards of one are not necessarily appropriate to the other.

Adherence to the observed practices of comparable businesses possibly guarantees no more than the avoidance of being atypical in so far as capital structure is concerned.

Traditionally, the firm's debt capacity and ability to repay has been measured in terms of balance sheet relationships and income statement data.

Figure 1
Conventional debt ratios

(i) Debt ratio: $\frac{\text{Total debt}}{\text{Total assets}}$

(ii) Long-term debt to total capitalization:

$$\frac{\text{long-term debt}}{\text{long-term debt} + \text{preference stock} + \text{ordinary stock}}$$

(iii) Times interest earned:

$$\frac{\text{profit before taxes} + \text{interest charges}}{\text{interest charges}}$$

Debt capacity is commonly expressed in terms of long-term debt/total of all long-term sources (total capitalisation), or long-term debt/total assets; 'times interest earned' or the ratio of net income available for debt servicing to the total amount of annual interest, expresses the limits of long-term borrowing. The more obvious weaknesses of these relationships should be borne in mind:

– there may be substantial changes in asset values, particularly as regards stock valuation and depreciation policies translating into changes in the debt to total assets ratio which have no bearing on the capacity to meet fixed cash drains.

– the net earnings figure found in the income statement is not necessarily akin to net cash inflow – an assumption which is implicit in the times interest earned ratio.

– the question of what the proper ratio is between earnings and debt servicing is problematic. Rather than measure risk it would seem to reflect the borrower's attitude toward risk bearing.

A more useful approach

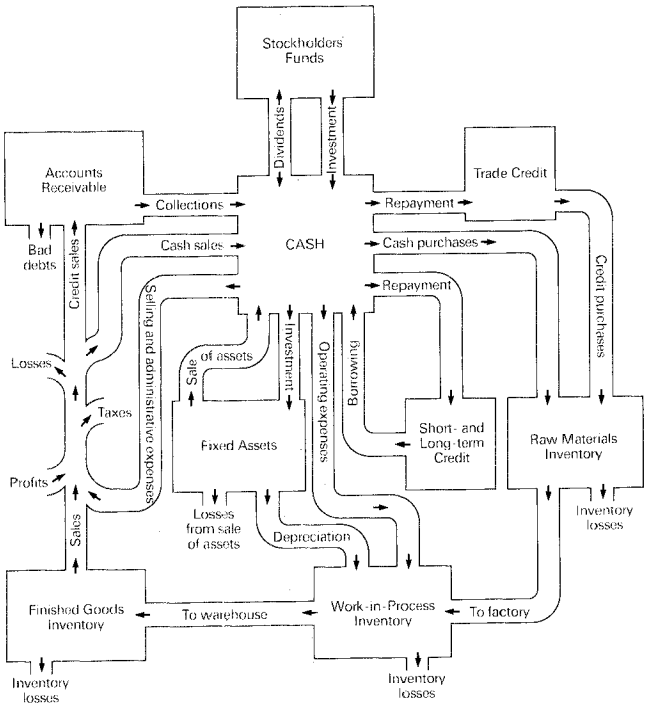
As stated earlier, the basic question in the apprisal of the magnitude of risk associated with long-term debt can be stated as: What are the chances of the business running out of cash in the forseeable future? Moreover, considering the problem from the viewpoint of a normally profitable

and reasonably well managed company, it is fair to say that the primary concern is with what might happen during a general recession when demand and, hence, profits are depressed.

The approach must therefore concern itself not with balance sheet or income statement ratios but directly with the factors influencing levels of cash inflow and outflow.

There are a number of different approaches to cash analysis. At the outset, it is important to identity the primary factors which produce major changes in cash flow. In particular, those that produce contractions in cash flow. The most significant factor will be sales volume. Other factors include cash expenditures for raw materials, the working relationship between finished goods on hand, work in progress and raw materials ordering, etc.

Figure 2
Complete cash flow of a hypothetical manufacturing firm



(Adapted from Techinques of Financial Analysis, 3rd edition Homewood I11 Richard D. Irwin, Inc., 1972)

Given the factors affecting cash flow, the next step is to observe their behaviour over time and in particular during conditions of depressed demand. Probably past experience will suggest a range of recession behaviour. Translating this in terms of the minimum rands of net inflow (or maximum rands of net outflow), period by period, it is possible to determine whether the company would become insolvent under such adverse conditions and, if so, how soon and by how much. This calculation will, in itself, give management some 'feel' for the nearness or remoteness of the event of cash insolvency. Should the company, even under adverse assumptions, still have a positive cash balance, this amount could be decided upon to represent the total amount of incremental fixed cash charges which the company could rely upon. Furthermore, given the nature and terms of a debt contract, the figure could be translated into the principal amount of additional debt the firm could assume with safety.

Problems to overcome

The relative simplicity suggested in this analysis is not intended to hide or avoid the complexities one encounters in practice. This analysis requires the guidance of someone well acquainted in financial analysis in order to steer the study of cash flows around potential pitfalls. The problems centre mainly on the accurate description of adjustment patterns of cash flows over time and the assessment of interdependence of factors affecting cash flows. Also, past slump periods may not have provided enough experience of the behaviour of sales, inventory levels, patterns of debtors' payments, etc. on which to base firm estimates of future behaviour.

Nevertheless, the criterion is derived entirely from within and is independent of external judgement or rules of thumb. In this respect it is both meaningful and useful in practice.

References

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