INTERNATIONAL TELECOMMUNICATION UNION

CCITT

E.524 (rev.1)

THE INTERNATIONAL
TELEGRAPH AND TELEPHONE
CONSULTATIVE COMMITTEE

TELEPHONE NETWORK AND ISDN

**QUALITY OF SERVICE,
NETWORK MANAGEMENT AND TRAFFIC
ENGINEERING**

OVERFLOW APPROXIMATIONS FOR NON-RANDOM INPUTS

Recommendation E.524 (rev.1)

Geneva, 1992

FOREWORD

The CCITT (the International Telegraph and Telephone Consultative Committee) is a permanent organ of the International Telecommunication Union (ITU). CCITT is responsible for studying technical, operating and tariff questions and issuing Recommendations on them with a view to standardizing telecommunications on a worldwide basis.

The Plenary Assembly of CCITT which meets every four years, establishes the topics for study and approves Recommendations prepared by its Study Groups. The approval of Recommendations by the members of CCITT between Plenary Assemblies is covered by the procedure laid down in CCITT Resolution No. 2 (Melbourne, 1988).

Recommendation E.524 was prepared by Study Group II and was approved under the Resolution No. 2 procedure on the 16th of June 1992.

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CCITT NOTE

In this Recommendation, the expression "Administration" is used for conciseness to indicate both a telecommunication Administration and a recognized private operating agency.

OVERFLOW APPROXIMATIONS FOR NON-RANDOM INPUTS

(revised 1992)

1 Introduction

This Recommendation introduces approximate methods for the calculation of blocking probabilities for individual traffic streams in a circuit group arrangement. It is based on contributions submitted in the Study Period 1984-1988 and is expected to be amended and expanded in the future (by adding the latest developments of methods).

The considered methods are necessary complements to those included in the existing Recommendation E.521 when it is required to take into account concepts such as cluster engineering with service equalization, service protection and end-to-end grade of service. Recommendation E.521 is then insufficient as it is concerned with the grade of service for only one non-random traffic stream in a circuit group.

Design methods concerning the above-mentioned areas are subject to further study and this Recommendation will serve as a reference when, in the future, Recommendation E.521 is complemented or replaced.

In this Recommendation the proposed methods are evaluated in terms of accuracy, processing time, memory requirements and programming effort. Other criteria may be relevant and added in the future.

The proposed methods are described briefly in § 2. Section 3 defines a set of examples of circuit group arrangements with exactly calculated (exact resolution of equations of state) individual blocking probabilities, to which the result of the methods can be compared. This leads to Table 2/E.524, where for each method the important criteria are listed. The publications cited in the reference section at the end contain detailed information about the mathematical background of each of the methods.

2 Proposed methods

The following methods are considered:

a) interrupted Poisson process (IPP) method;

b) equivalent capacity (EC) method;

c) approximative Wilkinson Wallström (AWW) method.

2.1 *IPP method*

IPP (Interrupted Poisson Process) method is a Poisson process interrupted by a random switch. The on-/off-duration of the random switch has a negative exponential distribution. Overflow traffic from a circuit group can be accurately approximated by an IPP, since IPP can represent bulk characteristics of overflow traffic. IPP has three parameters, namely, on-period intensity and mean on-/off-period durations. To approximate overflow traffic by an IPP, those three parameters are determined so that some moments of overflow traffic will coincide with those of IPP.

The following two kinds of moment match methods are considered in this Recommendation:

– three-moment match method [1] where IPP parameters are determined so that the first three moments of IPP will coincide with those of overflow traffic;

– four-moment ratio match method [2] where IPP parameters are determined so that the first moment and the ratios of the 2nd/3rd and 7th/8th binomial moments of IPP will coincide with those of overflow traffic.

To analyze a circuit group where multiple Poisson and overflow traffic streams are simultaneously offered, each overflow stream is approximated by an IPP. The IPP method is well suited to computer calculation. State transition equations of the circuit group with IPP inputs can be solved directly and no introduction of equivalent models is necessary. Characteristics of overflow traffic can be obtained from the solution of state transition equations. The main feature of the IPP method is that the individual means and variances of the overflow traffic can be solved.

2.2 *EC method*

The EC (equivalent capacity) method [3] does not use the traffic-moments but the transitional behaviour of the primary traffic, by introducing a certain function $\rho(n)$ versus the equivalent capacity (n) of the partial overflow traffic, as defined by the recurrent process:

$$\left[\begin{array}{l} \rho(o) = Em\,(\alpha) \quad \text{[Erlang loss formula]} \\ \dfrac{n}{\rho(n)} = (m + n - a) + \alpha \cdot \rho(n-1) \end{array}\right. \tag{2-1}$$

if n is a positive integer and approximated by linear interpolation, if not.

A practical approximation, considering the predominant overflow congestion states only, leads to the equations:

$$\frac{n_i}{n} = \frac{a_i\, \rho_i(n_i)/D_i\,(n_i + 1)}{\sum\limits_{k=1}^{x} a_k\, \rho_k(n_k)/D_k\,(n_k + 1)} \tag{2-2}$$

with:

$$D_i(n) = 1 + a_i\,[\rho_i(n) - \rho_i(n-1)] \tag{2-3}$$

defining the equivalent capacity (n_i) of the partial overflow traffic labelled i, and influenced by the mutual dependency between the partial overflow traffic streams.

The mean value of the partial second overflow is:

$$O_i = a_i \, \pi \, \rho_i(n_i) \tag{2-4}$$

where π is the time congestion of the overflow group.

The partial GOS (grade of service) equalization is fulfilled if:

$$\rho_i \, (n_i) = C \tag{2-5}$$

C being a constant to be chosen.

2.3 *AWW method*

The AWW (approximative Wilkinson Wallström) method uses an approximate ERT (equivalent random traffic) model based on an improvement of Rapp's approximation. The total overflow in traffic is split up in the individual parts by a simple expression, see equations (2-7) and (2-9). To calculate the total overflow traffic, any method can be used. An approximate Erlang formula calculation for which the speed is independent of the size of the calculated circuit group, is given in [4].

The following notations are used:

M	mean of total offered traffic;
V	variance of total offered traffic;
Z	V/M;
B	mean blocking of the studied group;
m_i, v_i, z_i, b_i	corresponding quantities for an individual traffic stream;
~	is used for overflow quantities.

2.3.1 *Blocking of overflow traffic*

For overflow calculations, an approximate ERT-model is used. By numerical investigations, a considerable improvement has been found to Rapp's classical approximation for the fictitious traffic. The error added by the approximation is small compared to the error of the ERT-model. It is known that ERT underestimates low blockings when mixing traffic of diverse peakedness [2]. The formula, which was given in [4] (although with one printing error), is for $Z > 1$:

$$A^* \approx V + Z(Z - 1)\,(2 + \gamma^{\beta})$$

where

$$\gamma = (2.36\,Z - 2.17)\,\log\,\{1 + (Z - 1)/[M(Z + 1.5)]\}$$

and

$$\beta = Z/(1.5\,M + 2\,Z - 1.3) \tag{2-6}$$

2.3.2 *Wallström formula for individual blocking*

There has been much interest in finding a simple and accurate formula for the individual blocked traffic $\tilde{}, m_i$. Already in 1967, Katz [5] proposed a formula of the type:

$$\tilde{m}_i = m_i B(1 - w + w z_i / Z) \tag{2-7}$$

with w being a suitable expression. Wallström proposed a very simple one but with reasonable results [6], [2]:

$$w = 1 - B \tag{2-8}$$

One practical problem is, however, that a small peaked substream could have a blocking $b_i > 1$ with this formula. To avoid such unreasonable results, a modification is used in this case. Let $z_{\max}$ be the largest individual z_i. Then the value used is:

$$w = \begin{cases} 1 - B & \text{if } z_{\max} < Z(1 + B)/B \\ Z(1 - B)/(B(z_{\max} - Z)) & \text{otherwise} \end{cases} \tag{2-9}$$

2.3.3 *Handling of overflow variances*

For the calculation of a large network it would be very cumbersome to keep track of all covariances. The normal case is that the overflow traffic from one trunk group is either lost or is offered to a secondary group without splitting up. Therefore, it is practical to include covariances in the individual overflow parameters $\tilde{v}_i$ so that they sum up to the total variance. The quantities $\tilde{v}_i$ are obtained from the total overflow variance $\tilde{V}$ by a simple splitting formula:

$$\tilde{v}_i = \tilde{V}_i v_i / V \tag{2-10}$$

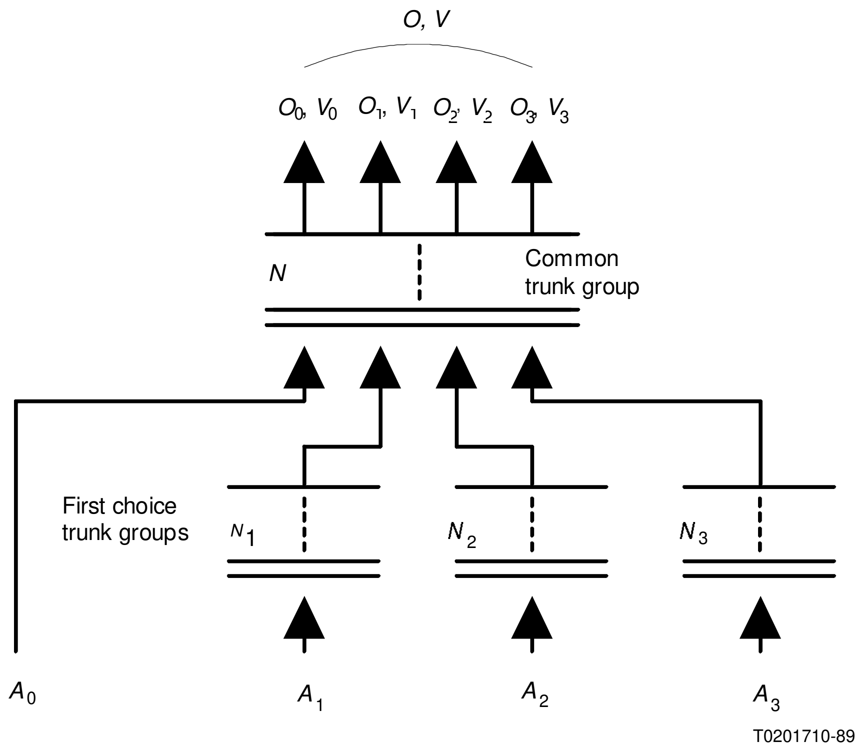
One can prove that Wallström's splitting formula (2-8) and formula (2-10) together with the ERT-model satisfies a certain consistency requirement. One will obtain the same values for the individual blocked traffic when calculating a circuit group of $N_1 + N_2$ circuits as when calculating first the N_1 circuits and then offering the overflow to the N_2 circuits.

Since the individual variances are treated in this manner, they are not comparable with the results reported in Table 2/E.524.

3 Examples and criteria for comparison

The defined methods are tested by calculating the examples given in Table 1/E.524.

The calculation model is given in Figure 1/E.524.



A_i Offered Poisson traffic volume
N_i Number of first choice trunk group
N Number of common trunk group
O_i Mean of individual overflow traffic from common trunk group
V_i Variance of individual overflow traffic from common trunk group
O Mean of total overflow traffic from common trunk group
V Variance of total overflow traffic from common trunk group

FIGURE 1/E.524

Calculation model

For comparison, the following criteria are established:

i) *Overflow traffic error*

– *accuracy of the individual overflow traffic mean and variance*

– *Mean error*

$$\varepsilon_O^I = \frac{\sum\limits_{\substack{\text{all} \\ \text{examples}}} \sum\limits_{i=1}^{M} \delta_O^i}{\{\text{number of streams}\}}$$

$$\varepsilon_V^I = \frac{\sum\limits_{\substack{\text{all} \\ \text{examples}}} \sum\limits_{i=1}^{M} \delta_V^i}{\{\text{number of streams}\}}$$

– *Standard deviation of error*

$$sd_O^I = \frac{\left\{ \sum\limits_{\substack{\text{all}\\ \text{examples}}} \sum\limits_{i=0}^{M} \left(\delta_O^i - \varepsilon_O^I\right)^2 \right\}^{1/2}}{\{\text{number of streams}\}}$$

$$sd_V^I = \frac{\left\{ \sum\limits_{\substack{\text{all}\\ \text{examples}}} \sum\limits_{i=0}^{M} \left(\delta_V^i - \varepsilon_V^I\right)^2 \right\}^{1/2}}{\{\text{number of streams}\}}$$

where

$$\delta^{i,O} = (O_i - \bar{O}_i)/\bar{O}_i$$

$$\delta^{i,V} = (V_i - \bar{V}_i)/\bar{V}_i$$

O_i, V_i : Calculated individual mean and variance of approximate method.

$\bar{O}_i$, $\bar{V}_i$: Exactly calculated individual mean and variance.

M : Number of input stream to common trunk group.

– *accuracy of the total overflow traffic mean and variance*

– *Mean error*

$$\varepsilon_O^T = \frac{\sum\limits_{\substack{\text{all}\\ \text{examples}}} \delta_O}{\{\text{number of examples}\}}$$

$$\varepsilon_V^T = \frac{\sum\limits_{\substack{\text{all}\\ \text{examples}}} \delta_V}{\{\text{number of examples}\}}$$

– *Standard deviation of error*

$$sd_O^T = \frac{\left\{ \sum\limits_{\substack{\text{all}\\ \text{examples}}} (\delta_O - \varepsilon_O^T)^2 \right\}^{1/2}}{\{\text{number of examples}\}}$$

$$sd_V^T = \frac{\left\{ \sum\limits_{\substack{\text{all}\\ \text{examples}}} (\delta_V - \varepsilon_V^T)^2 \right\}^{1/2}}{\{\text{number of examples}\}}$$

where

$$\delta_O = (O - \bar{O})/\bar{O}$$

$$\delta_V = (V - \bar{V})/\bar{V}$$

O, V : Calculated total mean and variance of approximate method.

$\bar{O}$; $\bar{V}$: Exactly calculated total mean and variance.

ii) *Computational effort*

– *Relative processor time*

$$C = \frac{\{\text{Total central processing unit (CPU) time for calculating all examples by using the approximate method}\}}{\{\text{Total CPU time for calculating all examples by Erland formula}\}}$$

– *Memory requirements*

$$M = \frac{\text{The size of memory required for the execution of approximative method}}{\text{The size of memory required for execution of Erland formula}}$$

– *Program size*

$$S = \frac{\text{Source program size for approximative method}}{\text{Source program size for Erlang formula}}$$

Note 1 – C, M and S, for a specific approximative method, should be based on the same processor, language and supporting algorithms.

Note 2 – Depending on the type of approximative method (direct calculating or recursive) different trade-offs between C, M and S may be reached, e.g. more memory versus less time, small program versus more time, etc.

TABLE 1a/E.524

**Exactly calculated mean and variance of individual overflow traffic –
Three first choice groups**

Case	A_1	A_2	A_3	A_0	O_0	O_1	O_2	O_3	O
	N_1	N_2	N_3	N	V_0	V_1	V_2	V_3	V
1	7.036	26.688	64.169	–	–	0.4337	0.7490	1.091	2.274
	5	28	70	11	–	0.7656	2.111	4.441	10.51
2	7.036	26.688	64.169	–	–	0.1449	0.2758	0.4944	0.9150
	5	28	70	16	–	0.2436	0.7328	1.911	4.293
3	7.036	26.688	64.169	–	–	0.01369	0.02846	0.06628	0.1084
	5	28	70	25	–	0.02041	0.06461	0.2205	0.4464
4	7.036	10.176	13.250	–	–	0.7459	1.262	1.785	3.792
	5	6	7	14	–	1.193	2.292	3.625	11.28
5	7.036	10.176	13.250	–	–	0.2888	0.4857	0.6832	1.458
	5	6	7	19	–	0.4636	0.9089	1.460	4.754
6	7.036	10.176	13.250	–	–	0.03570	0.05915	0.08237	0.1772
	5	6	7	26	–	0.05358	0.1026	0.1621	0.5249
7	7.036	32.395	77.617	–	–	0.4516	1.176	2.344	3.972
	5	31	77	16	–	0.7434	3.466	10.39	21.41
8	7.036	32.395	77.617	–	–	0.1538	0.4294	0.9739	1.557
	5	31	77	23	–	0.2427	1.200	4.219	8.558
9	7.036	32.395	77.617	–	–	0.01303	0.03984	0.1006	0.1535
	5	31	77	35	–	0.01841	0.09378	0.3690	0.7124
10	64.169	32.395	13.250	–	–	1.157	1.455	1.320	3.933
	70	31	7	15	–	4.442	4.256	2.849	18.28
11	64.169	32.395	13.250	–	–	0.5564	0.5849	0.4749	1.616
	70	31	7	21	–	2.026	1.675	1.023	8.008
12	64.169	32.395	13.250	–	–	0.06907	0.05265	0.03848	0.1602
	70	31	7	32	–	0.2167	0.1295	0.07165	0.7075
13	7.036	26.688	64.169	3.000	0.4064	0.5038	0.8274	1.160	2.897
	5	28	70	13	0.5578	0.8566	2.243	4.574	13.16
14	7.036	26.688	64.169	3.000	0.1460	0.1840	0.3384	0.5729	1.241
	5	28	70	18	0.1992	0.3043	0.8779	2.163	5.893
15	7.036	26.688	64.169	3.000	0.01170	0.01506	0.03086	0.07035	0.1280
	5	28	70	28	0.01472	0.02218	0.06861	0.2287	0.5374
16	7.036	32.395	77.617	1.000	0.1253	0.4451	1.156	2.304	4.031
	5	31	77	17	0.1392	0.7266	3.366	10.10	21.77
17	7.036	32.395	77.617	1.000	0.04250	0.1536	0.4275	0.9674	1.591
	5	31	77	24	0.04696	0.2409	1.183	4.148	8.775
18	7.036	32.395	77.617	1.000	0.004542	0.01687	0.05106	0.1282	0.2006
	5	31	77	35	0.004891	0.02398	0.1214	0.4751	0.9548
19	64.169	32.395	13.250	9.000	1.761	1.251	1.654	1.630	6.295
	70	31	7	21	3.052	4.517	4.406	3.103	27.73
20	64.169	32.395	13.250	9.000	0.6761	0.6501	0.7389	0.6427	2.708
	70	31	7	28	1.253	2.225	1.956	1.279	13.61
21	64.169	32.395	13.250	9.000	0.06219	0.09577	0.07978	0.06069	0.2984
	70	31	7	40	0.1054	0.2884	0.1887	0.1099	1.409

TABLE 1b/E.524

Exactly calculated mean and variance of individual overflow traffic – Two first choice groups

Case	A_1	A_2	A_3	A_0	O_0	O_1	O_2	O_3	O
	N_1	N_2	N_3	N	V_0	V_1	V_2	V_3	V
22	8.200	30.000	–	–	–	0.6153	1-139	–	1.755
	5	30	–	10	–	1.179	3.473	–	6.159
23	8.200	30.000	–	–	–	1.807	2.465	–	4.272
	5	30	–	5	–	3.263	7.431	–	13.00
24	8.200	30.000	–	–	–	0.01866	0.04814	–	0.06680
	5	30	–	21	–	0.03026	0.1233	–	0.1993
25	8.200	30.000	–	–	–	0.2111	0.4629	–	0.6740
	5	30	–	14	–	0.3902	1.372	–	2.355
26	8.200	14.300	–	–	–	0.04699	0.09279	–	0.1398
	5	7	–	22	–	0.07700	0.1980	–	0.3724
27	8.200	14.300	–	–	–	0.3744	0.7547	–	1.129
	5	7	–	16	–	0.6603	1.763	–	3.322
28	8.200	14.300	–	–	–	0.9282	1.892	–	2.820
	5	7	–	12	–	1.614	4.212	–	7.774
29	8.200	14.300	–	–	–	2.002	4.095	–	6.098
	5	7	–	7	–	3.272	7.806	–	13.64
30	8.200	42.000	–	–	–	0.02324	0.09886	–	0.1221
	5	37	–	27	–	0.03602	0.3019	–	0.4197
31	8.200	42.000	–	–	–	0.2136	0.8353	–	1.049
	5	37	–	19	–	0.3682	2.945	–	4.204
32	8.200	42.000	–	–	–	1.499	4.437	–	5.935
	5	37	–	8	–	2.616	14.60	–	21.00
33	8.200	42.000	–	–	–	0.6940	2.416	–	3.110
	5	37	–	13	–	1.237	8.493	–	12.30
34	30.000	14.300	–	–	–	0.06570	0.05450	–	0.1202
	30	7	–	25	–	0.1628	0.1116	–	0.3922
35	30.000	14.300	–	–	–	0.4669	0.4662	–	0.9331
	30	7	–	18	–	1.300	1.088	–	3.461
36	30.000	14.300	–	–	–	1.374	1.739	–	3.113
	30	7	–	12	–	3.932	4.001	–	10.91
37	30.000	14.300	–	–	–	2.425	3.806	–	6.231
	30	7	–	7	–	6.994	7.628	–	18.16
38	8.200	67.900	–	–	–	0.01656	0.1007	–	0.1173
	5	65	–	30	–	0.02497	0.3667	–	0.4658
39	8.200	67.900	–	–	–	0.1835	0.9716	–	1.155
	5	65	–	20	–	0.3132	4.189	–	5.488
40	8.200	67.900	–	–	–	0.5393	2.449	–	2.986
	5	65	–	14	–	0.9685	10.73	–	14.35
41	8.200	67.900	–	–	–	1.361	4.707	–	6.068
	5	65	–	8	–	2.441	19.71	–	26.55
42	51.500	14.300	–	–	–	0.07517	0.04089	–	0.1161
	54	7	–	27	–	0.2290	0.08146	–	0.4339
43	51.500	14.300	–	–	–	0.6402	0.4689	–	1.109
	54	7	–	18	–	2.248	1.101	–	4.799

TABLE 1c/E.524

Exactly calculated mean and variance of individual overflow traffic – One first choice group

Case	A_1	A_2	A_3	A_0	O_0	O_1	O_2	O_3	O
	N_1	N_2	N_3	N	V_0	V_1	V_2	V_3	V
44	51.500	14.300	–	–	–	1.403	1.362	–	2.765
	54	7	–	13	–	5.079	3.224	–	11.50
45	51.500	14.300	–	–	–	2.586	3.675	–	6.261
	54	7	–	7	–	9.612	7.513	–	21.11
46	8.200	–	–	4.000	0.03309	0.04990	–	–	0.08299
	5	–	–	16	0.04785	0.08712	–	–	0.1789
47	8.200	–	–	4.000	0.3494	0.4859	–	–	0.8354
	5	–	–	11	0.5382	0.9155	–	–	1.975
48	8.200	–	–	4.000	0.9011	1.169	–	–	2.070
	5	–	–	8	1.327	2.120	–	–	4.554
49	8.200	–	–	4.000	1.802	2.142	–	–	3.944
	5	–	–	5	2.369	3.588	–	–	7.333
50	30.000	–	–	4.000	0.01660	0.05973	–	–	0.07633
	30	–	–	20	0.02296	0.1558	–	–	0.2228
51	30.000	–	–	4.000	0.1991	0.5806	–	–	0.7796
	30	–	–	13	0.3062	1.743	–	–	2.656
52	30.000	–	–	4.000	0.5988	1.400	–	–	1.999
	30	–	–	9	0.9338	4.255	–	–	6.730
53	30.000	–	–	4.000	1.560	2.558	–	–	4.118
	30	–	–	5	2.199	7.620	–	–	12.01
54	51.500	–	–	4.000	0.01445	0.07537	–	–	0.08982
	54	–	–	22	0.01966	0.2413	–	–	0.3131
55	51.500	–	–	4.000	0.1208	0.5143	–	–	0.6351
	54	–	–	15	0.1819	1.893	–	–	2.575
56	51.500	–	–	4.000	0.4286	1.383	–	–	1.812
	54	–	–	10	0.6788	5.300	–	–	7.549
57	51.500	–	–	4.000	1.145	2.429	–	–	3.574
	54	–	–	6	1.726	9.299	–	–	13.55

4 Summary of results

The available methods and the performance measures with respect to the criteria are listed in Table 2/E.524.

TABLE 2/E.524

Comparison of different approximation methods

Functions	Comparison		
	Computational effort		
Method	Processor time	Memory requirements	Programming effort
IPP method			
3 moment match	400	5	50
4 moment ratio	1000	5	50
EC method	40	2	3
AWW method	3	1.6	3

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