All drawings appearing in this Recommendation have been done in Autocad.

Recommendation E.524

xe ""§OVERFLOW APPROXIMATIONS FOR NON–RANDOM INPUTS

1 Introduction

This Recommendation introduces approximate methods for the calculation of blocking probabilitiesxe " blocking probabilities"§ for individual traffic streams in a circuit group arrangement. It is based on contributions submitted in the Study Period 1984–1988 and is expected to be amended and expanded in the future (by adding the latest developments of methods).

The considered methods are necessary complements to those included in the existing Recommendation E.521 when it is required to take into account concepts such as cluster engineering with service equalization, service protection and end–to–end grade of service. Recommendation E.521 is then insufficient as it is concerned with the grade of service for only one non–random traffic stream in a circuit group.

Design methods concerning the above–mentioned areas are subject to further study and this Recommendation will serve as a reference when, in the future, Recommendation E.521 is complemented or replaced.

In this Recommendation the proposed methods are evaluated in terms of accuracy, processing time, memory requirements and programming effort. Other criteria may be relevant and added in the future.

The proposed methods are described briefly in § 2. Section 3 defines a set of examples of circuit group arrangements with exactly calculated (exact resolution of equations of state) individual blocking probabilities, to which the result of the methods can be compared. This leads to a table in § 4, where for each method the important criteria are listed. The publications cited in the reference section at the end contain detailed information about the mathematical background of each of the methods.

2 Proposed methods

The following methods are considered:

a) Interrupted Poisson Process (IPP) method,

b) Equivalent Capacity (EC) method,

c) Approximative Wilkinson Wallström (AWW) method.

2.1 *IPP method*

IPP (Interrupted Poisson Process)xe " (Interrupted Poisson Process)"§ is a Poisson process interrupted by a random switch. The on–/off–duration of the random switch has a negative exponential distribution. Overflow traffic from a circuit group can be accurately approximated by an IPP, since IPP can represent bulk characteristics of overflow traffic. IPP has three parameters, namely, on–period intensity and mean on–/off–period durations. To approximate overflow traffic by an IPP, those three parameters are determined so that some moments of overflow traffic will coincide with those of IPP.

The following two kinds of moment match methods are considered in this Recommendation:

– three–moment match method [1] – where IPP parameters are determined so that the first three moments of IPP will coincide with those of overflow traffic;

– four–moment ratio match method [2] where IPP parameters are determined so that the first moment and the ratios of the 2nd/3rd and 7th/8th binomial moments of IPP will coincide with those of overflow traffic.

To analyze a circuit group where multiple Poisson and overflow traffic streams are simultaneously offered, each overflow stream is approximated by an IPP. The IPP method is well suited to computer calculation. State transition equations of the circuit group with IPP inputs can be solved directly and no introduction of equivalent models is necessary. Characteristics of overflow traffic can be obtained from the solution of state transition equations. The main feature of the IPP method is that the individual means and variances of the overflow traffic can be solved.

2.2 *EC method*

The EC (Equivalent Capacity) methodxe " (Equivalent Capacity) method"§ [3] does not use the traffic–moments but the transitional behaviour of the primary traffic, by introducing a certain function r(*n*) versus the equivalent capacity (*n*) of the partial overflow traffic, as defined by the recurrent process:

(2–1)

if *n* is a positive integer and approximated by linear interpolation, if not.

A practical approximation, considering the predominant overflow congestion states only, leads to the equations:

(2–2)

with:

$$Di(n) = 1 + ai \qquad (2\text{–}3)$$

defining the equivalent capacity (*ni*) of the partial overflow traffic labelled *i*, and influenced by the mutual dependency between the partial overflow traffic streams.

The mean value of the partial second overflow is:

$$Oi = ai \text{ p } \mathrm{r}i(ni) \qquad (2\text{–}4)$$

where p is the time congestion of the overflow group.

The partial GOS (grade of service) equalization is fulfilled if:

$$\mathrm{r}i\ (ni) = C \qquad (2\text{–}5)$$

C being a constant to be chosen.

2.3 *AWW method*

The AWW (Approximative Wilkinson Wallström) methodxe " (Approximative Wilkinson Wallström) method"§ uses an approximate ERT (Equivalent Random Traffic) model based on an improvement of Rapp's approximation. The total overflow in traffic is split up in the individual parts by a simple expression, see Equations (2–7) and (2–9). To calculate the total overflow traffic, any method can be used. An approximate Erlang formula calculation for which the speed is independent of the size of the calculated circuit group is given in [4].

The following notations are used:

M mean of total offered traffic;

V variance of total offered traffic;

Z V/M;

B mean blocking of the studied group;

m_i, v_i, z_i, b_i

~ is used for overflow quantities.

2.3.1 *Blocking of overflow traffic*

For overflow calculations, an approximate ERT–model is used. By numerical investigations, a considerable improvement has been found to Rapp's classical approximation for the fictitious traffic. The error added by the approximation is small compared to the error of the ERT–model. It is known that ERT underestimates low blockings when mixing traffic of diverse peakedness [2]. The formula, which was given in [4] (although with one printing error), is for $Z > 1$:

$$A^* \quad V + Z(Z - 1)(2 + g\beta)$$

where

$$g = (2.36\, Z - 2.17) \log \{1 + (z - 1)/[M(Z + 1.5)]\}$$

and

$$\beta = Z/(1.5M + 2Z - 1.3) \tag{2–6}$$

2.3.2 *Wallström formula for individual blocking*

There has been much interest in finding a simple and accurate formula for the individual blocked traffic m£i. Already in 1967, Katz [5] proposed a formula of the type

\s\up4(~) (2–7)

with w being a suitable expression. Wallström proposed a very simple one but with reasonable results [6], [2]:

$$w = 1 - B \tag{2–8}$$

One practical problem is, however, that a small peaked substream could have a blocking $b_i > 1$ with this formula. To avoid such unreasonable results a modification is used in this case. Let z_{max} be the largest individual z_i.

Then the value used is

$w =$ (2–9)

2.3.3 *Handling of overflow variances*

For the calculation of a large network it would be very cumbersome to keep track of all covariances. The normal case is that the overflow traffic from one trunk group is either lost or is offered to a secondary group without splitting up. Therefore it is practical to include covariances in the individual overflow parameters \s\up4(~) so that they sum up to the total variance. The quantities *vi* are obtained from the total overflow variance \s\up6(~) by a simple splitting formula:

\s\up4(~) (2–10)

One can prove that Wallström's splitting formula (2–8) and formula (2–10) together with the ERT–model satisfies a certain consistency requirement. One will obtain the same values for the individual blocked traffic when calculating a circuit group of $N1 + N2$ circuits as when calculating first the $N1$ circuits and then offering the overflow to the $N2$ circuits.

Since the individual variances are treated in this manner, they are not comparable with the results reported in Table 2/E.524.

3 Examples and criteria for comparison

The defined methods are tested by calculating the examples given in Table 1/E.524.

The calculation model is given in Figure 1/E.524.

For comparison, the following criteria are established:

– accuracy of the overflow traffic mean and variance (mean and standard deviation),

– computational criteria (processor time, memory requirements, programming effort).

Figure 1/E.524 - T0200630-87

TABLE 1a/E.524

Exactly calculated mean and variance of individual overflow traffic – Three first choice circuit groups

Case

$A1$

$A2$

*A*3

a1

a2

a3

*A*0

N

*O*0

*O*1

*O*2

*O*3

*N*1

*N*2

*N*3

*Z*1

*Z*2

*Z*3

*V*0

*V*1

*V*2

*V*3

1
7.036
26.688
64.169
3.003
3.001
3.000

–
0.4337
0.7490
1.091

5
28
70
1.573
3.022
4.527
–
11
–
0.7656
2.110
4.441

2

7.036

26.688

64.169

3.003

3.001

3.000

–

0.1149

0.2758

0.4944

5

28

70

1.573

3.022

4.527

–

16

–

0.2436

0.7328

1.911

3

7.036

26.688

64.169

3.003

3.001

3.000

–

0.01369

0.02846

0.06627

5

28

70

1.573

3.022

4.527

–

25

–

0.02041

0.06461

0.2205

4

7.036

10.176

13.250

3.003

5.003

7.002

–

0.7459

1.262

1.785

5

6

7

1.573

1.567

1.559

–

14

–

1.193

2.292

3.624

5

7.036

10.176

13.250

3.003

5.003

7.002

–

0.2884

0.4857

0.6832

5

6

7

1.573

1.567

1.559

–

19

–

0.4636

0.9089

1.460

6

7.036

10.176

13.250

3.003

5.003

7.002

–

0.03570

0.05915

0.08237

5

6

7

1.573

1.567

1.559

–

26

–

0.05358

0.1026

0.1621

7

7.036

32.395

77.617

3.003

5.002

7.001

–

0.4516

1.176

2.344

5

31

77

1.573

3.029

4.511

–

16

–

0.7434

3.466

10.39

8

7.036

32.395

77.617

3.003

5.002

7.001

–

0.1538

0.4294

0.9739

5

31

77

1.573

3.029

4.511

–

23

–

0.2427

1.200

4.219

9

7.036

32.395

77.617

3.003

5.002

7.001

–

0.01303

0.03984

0.1006

5

31

77

1.573

3.029

4.511

–

35

–

0.1841

0.09378

0.3690

10

64.169

32.395

13.250

3.000

5.002

7.002

–

1.157

1.456

1.320

70

31

7

4.527

3.029

1.559

–

15

–

4.442

4.256

2.850

11

64.169

32.395

13.250

3.000

5.002

7.002

–

0.5564

0.5849

0.4749

70

31

7

4.527

3.029

1.559

–

21

–

2.026

1.675

1.023

12

64.169

32.395

13.250

3.000

5.002

7.002

–

0.06907

0.05265

0.03848

70

31

7

4.527

3.029

1.559

–

32

–

0.2167

0.1295

0.07165

13

7.036

26.688

64.169

3.003

3.001

3.000

0.4064

0.5038

0.8274

1.160

5

28

70

1.573

3.022

4.527

3.000

13

0.5578

0.8566

2.243

4.574

14

7.036

26.688

64.169

3.003

3.001

3.000

0.1460

0.1840

0.3384

0.5729

5

28

70

1.573

3.022

4.527

3.000

18

0.1992

0.3043

0.8779

2.163

TABLE 1a/E.524 *(cont.)*

Case
*A*1
*A*2
*A*3
a1
a2
a3
*A*0
N
*O*0
*O*1
*O*2
*O*3
*N*1
*N*2
*N*3
*Z*1
*Z*2
*Z*3
*V*0

$V1$

$V2$

$V3$

15

7.036

26.688

64.169

3.003

3.001

3.000

0.01170

0.01506

0.03086

0.07035

5

28

70

1.573

3.022

4.527

3.000

28

0.01472

0.02218

0.06861

0.2287

16

7.036

32.395

77.617

3.003

5.002

7.001

0.1253

0.4451

1.156

2.304

5

31

77

1.573

3.029

4.511

1.000

17

0.1392

0.7266

3.366

10.10

17

7.036

32.395

77.617

3.003

5.002

7.001

0.04250

0.1536

0.4275

0.9674

5

31

77

1.573

3.029

4.511

1.000

24

0.04696

0.2409

1.183

4.148

18

7.036

32.395

77.617

3.003

5.002

7.001

0.004542

0.01687

0.05106

0.1282

5

31

77

1.573

3.029

4.511

1.000

35

0.004891

0.02398

0.1214

0.4751

19

64.169

32.395

13.250

3.000

5.002

7.002

1.761

1.251

1.654

1.630

70

31

7

4.527

3.029

1.559

9.000

21

3.052

4.517

4.406

3.103

20

64.169

32.395

13.250

3.000

5.002

7.002

0.6761

0.6501

0.7389

0.6427

70

31

7

4.527

3.029

1.559

9.000

28

1.253

2.225

1.956

1.279

21

64.169

32.395

13.250

3.000

5.002

7.002

0.06219

0.09577

0.07978

0.06069

70

31

7

4.527

3.029

1.559

9.000
40

0.1054

0.2884

0.1887

0.1099

TABLE 1b/E.524

Exactly calculated mean and variance of individual overflow traffic – Two first choice circuit groups

$A1$	$N1$	$A2$	$N2$	N	$O1$	$V1$	$O2$	$V2$
8.2	5	30.0	30	10	0.6155	1.1791	1.1393	3.4723

5
1.8068
3.2634
2.4656
7.4312

21
0.0188
0.0304
0.0485
0.1240

14
0.2108
0.3898
0.4624
1.3701

14.3
7
22
0.0470
0.0771

0.0929

0.1983

16

0.3743

0.6602

0.7546

1.7626

12

0.9282

1.6137

1.8320

4.2120

7

2.0023

3.2718

4.0953

7.8064

42.0

37

27

0.0230

0.0354

0.0978

0.2984

19
0.2136
0.3683
0.8356
2.9450

8
1.4984
2.6161
4.4363
14.6018

13

0.6940

1.2375

2.4148

8.4923

30.0

30

14.3

7

25

0.0653

0.1613

0.0541

0.1112

18
0.4664
1.2990
0.4662
1.0879

12
1.3746
3.9321
1.7390
4.0015

7

2.4255

6.9941

3.8063

7.6277

8.2

5

67.9

65

30

0.0160

0.0242

0.0979

0.3548

20

0.1839

0.3141

0.9739

4.1953

14
0.5385
0.9676
2.4438
10.7208

8
1.3598
1.4401
4.7035
19.7109

51.5
54
14.3
7
27
0.0735
0.2239
0.0399
0.0802

19
0.6404
1.2499
0.4699
1.1030

13
1.4033
5.0795
1.3609
3.2229

7
2.5873
9.6136
3.6744
7.5139

TABLE 1c/E.524

Exactly calculated mean and variance of individual overflow traffic – One first choice circuit group

*A*1	*N*1	*A*0	*N*	*O*1	*V*1	*O*0	*V*0
8.2	5	4.0	16	0.0499	0.0872	0.0331	0.0479

11
0.4859
0.9154
0.3494
0.5382

9
1.1692
2.1202
0.9011
1.3274

5
2.1422
3.5883
1.8018
2.3694

30.0
30

20
0.0601
0.1565

0.0167

0.023

13

0.5804

1.7427

0.1990

0.3062

9

1.3997

4.2546

0.5988

0.9338

5

2.5579

5.6196

1.5661

2.1991

51.5

54

22

0.9751

0.2497

0.0144

0.0197

15

0.5141

1.8924

0.1209

0.1819

10

1.8820

5.3004

0.4297

0.6790

5

2.4294

3.2974

1.1450

1.7255

4 Summary of results

The available methods and the performance measures with respect to the criteria are listed in Table 2/E.524.

TABLE 2/E.524

Comparison of different approximation methods

Functions

Input

Output

Comparison

Highest

Overflow traffic error

Computational effort

Required higher moments of Mean

Variance

Method

moments
overflow traffic

Mean

Standard deviation

Mean

Standard deviation

Processor time

Memory require-ments

Program-ming effort

IPP method

a) 3 moment match

3

3

– 0.0045

0.0585

– 0.0210

0.0922

b) 4 moment ratio

8

0.0008

0.0255

– 0.0053

0.0373

EC method

1

1

– 0.0661

0.1527

AWW method

2

2

– 0.0448

0.1647

References

[1] MATSUMOTO (J.) and WATANABE (Y.): Analysis of individual traffic characteristics for queuing systems with multiple Poisson and overflow inputs. *Proc. 10th ITC,* paper 5.3.1, Montreal, 1983.

[2] RENEBY (L.): On individual and overall losses in overflow systems. *Proc. 10th ITC,* paper 5.3.5, Montreal, 1983.

[3] LE GALL (P.): Overflow traffic combination and cluster engineering. *Proc. 11th ITC,*

paper 2.2B–1, Kyoto, 1985.

[4] LINDBERG (P.), NIVERT, (K.), SAGERHOLM, (B.): Economy and service aspects of different designs of alternate routing networks. *Proc. 11th ITC*, Kyoto, 1985.

[5] KATZ (S.): Statistical performance analysis of a switched communications network. *Proc. 5th ITC*, New York, 1967.

[6] LINDBERGER (K.): Simple approximations of overflow system quantities for additional demands in the optimization. *Proc. 10th ITC*, Montreal, 1983.