ANNEX B
(to Recommendation E.506)

Example using weighted least squares method

B.1 *Telex data*

The telex traffic between the following countries has been analyzed:

– Germany (D)
– Denmark (DNK)
– USA (USA)
– Finland (FIN)
– Norway (NOR)
– Sweden (S)

The data consists of yearly observations from 1973 to 1984 [19].

B.2 *Forecasting*

Before using the weighted least squares method, separate forecasts for the traffic matrix have to be made. In this example a simple ARIMA (0,2,1) model with logarithmic transformed observations without explanatory variables is used for forecasting. It may be possible to develop better forecasting models for the telex traffic between the various countries. However the main point in this example only is to illustrate the use of the weighted least squares technique.

Forecasts for 1984 based on observations from 1973 to 1983 are given in Table B–1/E.506.

TABLE B–1/E.506

Forecasts for telex traffic between Germany(D), Denmark(DNK), USA(USA), Finland(FIN), Norway(NOR) and Sweden(S) in 1984

From

D

DNK

USA

FIN

NOR

S

Sum

Forecasted

To

sum

D

–

4869

12 630

2879

2397

5230

28 005

27 788

DNK
5196
–

1655
751
1270
1959
10 831
10 805

USA
11 103
1313
–
719
1657
2401
17 193
17 009

FIN
2655
715
741
–
489
1896
6496
6458

NOR
2415
1255
1821
541
–
1548
7580
7597

S

4828

1821

2283

1798

1333

–

12 063

12 053

Sum

26 197

9973

19 130

6688

7146

13 034

Forecasted sum

26 097

9967

19 353

6659

7110

12 914

It should be noticed that there is no consistency between row and column sum forecasts and forecasts of the elements in the traffic matrix. For instance, the sum of forecasted outgoing telex traffic from Germany is 28 005, while the forecasted row sum is 27 788.

To adjust the forecasts to get consistency and to utilize both row/column forecasts and forecasts of the traffic elements the weighted least squares method is used.

B.3 *Adjustment of the traffic matrix forecasts*

To be able to use the weighted least squares method, the weights and the separate forecasts are needed as input. The separate forecasts are found in Table B–2/E.506, while the weights are based on the mean squared one step ahead forecasting errors.

Let *yt* be the traffic at time *t*. The ARIMA (0,2,1) model with logarithmic transformed data is given by:

$$zt = (1 - B)2 \ln yt = (1 - qB)\, at$$

or

$$zt = at - q at–1$$

where

$zt = \ln yt - 2 \ln yt–1 + \ln yt–2$

at is white noise,

q is a parameter,

B is the backwards shift operator.

The mean squared one step ahead forecasting error of *zt* is:

$$MSQ = \text{S}\ (zt - z¢t–1(1))2$$

where

\s\up4(^)*t*–1(1) is the one step ahead forecast.

The results of using the weighted least squares method is found in Table B–3/E.506 and show that the factors in Table B–1/E.506 have been adjusted. In this example only minor changes have been performed because of the high conformity in the forecasts of row/column sums and traffic elements.

TABLE B–2/E.506

Inverse weights as mean as squared one step ahead forecasting errors of telex traffic (100–4) between Germany(D), Denmark(DNK), USA(USA), Finland(FIN), Norway(NOR) and Sweden(S) in 1984

From

D

DNK

USA

FIN

NOR

S

Sum

To

D

–

28.72

13.18

11.40

8.29

44.61

7.77

DNK
5.91
–
43.14
18.28
39.99
18.40
10.61

USA
23.76
39.19
–
42.07
50.72
51.55
21.27

FIN
23.05
12.15
99.08
–
34.41
19.96
17.46

NOR
21.47
40.16
132.57
24.64
–
17.15
20.56

S

6.38

12.95

28.60

28.08

8.76

–

6.48

Sum

6.15

3.85

14.27

9.55

12.94

8.53

TABLE B–3/E.506

Adjusted telex forecasts using the weighted least squares method

From

D

DNK

USA

FIN

NOR

S

Sum

To

D

–

4850

12 684

2858

2383

5090

27 865

DNK
5185
–
1674
750
1257
1959
10 825

USA
11 001
1321
–
717
1644

2407
17 090

FIN
2633
715
745
–
487
1891
6471

NOR
2402
1258
1870
540
–
1547
7617

S

4823

1817

2307

1788

1331

–

12 066

Sum

26 044

9961

19 280

6653

7102

12 894