INTERNATIONAL TELECOMMUNICATION UNION

CCITT

E.501 (rev.1)

THE INTERNATIONAL
TELEGRAPH AND TELEPHONE
CONSULTATIVE COMMITTEE

TELEPHONE NETWORK AND ISDN

**QUALITY OF SERVICE,
NETWORK MANAGEMENT AND TRAFFIC
ENGINEERING**

ESTIMATION OF TRAFFIC OFFERED IN THE NETWORK

Recommendation E.501 (rev.1)

Geneva, 1992

FOREWORD

The CCITT (the International Telegraph and Telephone Consultative Committee) is a permanent organ of the International Telecommunication Union (ITU). CCITT is responsible for studying technical, operating and tariff questions and issuing Recommendations on them with a view to standardizing telecommunications on a worldwide basis.

The Plenary Assembly of CCITT which meets every four years, establishes the topics for study and approves Recommendations prepared by its Study Groups. The approval of Recommendations by the members of CCITT between Plenary Assemblies is covered by the procedure laid down in CCITT Resolution No. 2 (Melbourne, 1988).

Recommendation E.501 was prepared by Study Group II and was approved under the Resolution No. 2 procedure on the 16 of June 1992.

CCITT NOTE

In this Recommendation, the expression "Administration" is used for conciseness to indicate both a telecommunication Administration and a recognized private operating agency.

Recommendation E.501

ESTIMATION OF TRAFFIC OFFERED IN THE NETWORK

(revised 1992)

1 Introduction

For planning the growth of the network, the following quantities must be estimated from measurements:

- traffic offered to circuit groups;
- traffic offered to destinations, on a origin-destination basis;
- traffic offered to exchanges;
- call attempts offered to exchanges;
- traffic offered to signalling links.

These quantities are normally estimated from measurements of busy-hour carried traffic and call attempts, but there are a number of factors which may need to be taken into account within the measurement and estimation procedures:

a) Measurements may need to be subdivided, e.g. on a destination basis, or by call type (for example, calls using different signalling systems).

b) It may not be possible to obtain a complete record of traffic carried. For example, in a network with high usage and final groups, it may not be possible to measure the traffic overflowing from each high usage group.

c) Measurements may be affected by congestion. This will generally result in a decrease in traffic carried, but the decrease may be affected by customer's repeat attempts and by the actions (for example, automatic repeat attempts) of other network components.

d) When high levels of congestion persist for a lengthy period (many days), some customers may avoid making calls during the congested period of each day. This apparent missing component of offered traffic is known as suppressed traffic. It should be taken into account in planning since the offered traffic will increase when the equipment is augmented. At present, suitable algorithms for estimating suppressed traffic have not been defined.

Three situations should be distinguished:

i) congestion upstream of the measurement point – This is not directly observable;

ii) congestion due to the measured equipment – Congestion measurements should be used to detect this;

iii) congestion downstream of the measurement point – This can often be detected from measurements of ineffective traffic or completion ratio. Note that where groups are bothway, congestion elsewhere in the network may be both upstream and downstream of the measurement point for different parcels of traffic.

When congestion is due to the measured equipment, this must be properly accounted for in the estimation of traffic offered which is used for planning the growth of the measured equipment.

When congestion arises elsewhere in the network, the planner needs to consider whether or not the congestion will remain throughout the considered planning period. This may be difficult if he does not have control of the congested equipment.

This Recommendation presents estimation procedures for two of the situations described above. Sections 2 and 3 have the aim of the determination of traffic offered to circuit group and namely Section 2 deals with the estimation of traffic offered to a fully-operative only-route circuit group which may be in significant congestion. Section 3 deals with a high-usage and final group arrangement with no significant congestion. Section 4 provides a procedure to determine traffic offered to destinations on an origin-destination basis, when only measurements of traffic intensity on trunk groups are available or when direct measurements on origin-destination traffic offered are also available.

In Section 4 the estimated traffic offered is the "equivalent traffic offered" used in the pure lost call model as defined in Annex B, while in Sections 2 and 3 in the evaluation of traffic offered, the user's repeat attempts are taken into account.

These estimation procedures should be applied to individual busy-hour measurements. The resulting estimates of traffic offered in each hour should then be accumulated according to the procedures described in Recommendation E.500.

2 Only-route circuit group

2.1 *No significant congestion*

Traffic offered will equal traffic carried measured according to Recommendation E.500. No estimation is required.

2.2 *Significant congestion*

Let A_c be the *traffic carried* on the circuit group. Then, on the assumption that augmentation of the circuit group would have no effect on the mean holding time of calls carried or on the completion ratio of calls carried, the *traffic offered* to the circuit group may be expressed as

$$A = A_c \frac{(1 - WB)}{(1 - B)}$$

where B is the present average loss probability for all call attempts to the considered circuit group, and W is a parameter representing the effect of call repetitions. Models for W are presented in Annex A.

To facilitate the quick determination of offered traffic according to the approximate procedure in Annex A, Table A-1/E.501 including numerical values of the factor $(1 - WB)/(1 - B)$ was prepared for a wide range of B, H and r' (for the definition of H and r', see Annex A). For the use of Table A-1/E.501, see Note 2 in Annex A.

Note 1 – Annex A gives a derivation of this relationship, and also describes a more complex model which may be of use when measurements of completion ratios are available.

Note 2 – When measurements of completion ratios are not available a W value may be selected from the range 0.6 to 0.9. It should be noted that a lower value of W corresponds to a higher estimate of traffic offered. Administrations are encouraged to exchange the values of W that they propose to use.

Note 3 – Administrations should maintain records of data collected before and after augmentations of circuit groups. This data will enable a check on the validity of the above formula, and on the validity of the value of *W* used.

Note 4 – In order to apply this formula, it is normally assumed that the circuit group is in a fully operative condition or that any faulty circuits have been taken out of service. If faulty circuits or faulty transmission or signalling equipment associated with these circuits remain in service, then the formula may give incorrect results.

3 High-usage/final network arrangement

3.1 *High-usage group with no significant congestion on the final group*

3.1.1 Where a relation is served by a high-usage and final group arrangement, it is necessary to take simultaneous measurements on both circuit groups.

Let A_H be the traffic carried on the high-usage group, and A_F the traffic overflowing from this high-usage group and carried on the final group. With no significant congestion on the final group, the traffic offered to the high-usage group is:

$$A = A_H + A_F$$

3.1.2 Two distinct types of procedure are recommended, each with several possible approaches. The method given in § 3.1.2.1 a) is preferred because it is the most accurate, although it may be the most difficult to apply. The methods of § 3.1.2.2 may be used as additional estimates.

3.1.2.1 Simultaneous measurements are taken of A_H and the total traffic carried on the final group. Three methods are given for estimating A_F, in decreasing order of preference:

a) A_F is measured directly. In most circumstances this may be achieved by measuring traffic carried on the final group on a destination basis.

b) The total traffic carried on the final group is broken down by destination in proportion to the number of effective calls to each destination.

c) The traffic carried on the final group is broken down according to ratios between the bids from the high-usage groups and the total number of bids to the final group.

3.1.2.2 Two alternative methods are given for estimating the traffic offered to the high-usage group which, in this circumstance, equals the equivalent traffic offered:

a) *A* is estimated from the relationship

$$A_H = A\,[1 - E_N(A)]$$

here $E_N(A)$ is the Erlang loss formula, *N* is the number of working circuits on the high-usage group. The estimation may be made by an iterative computer program, or manually by the use of tables or graphs.

The accuracy of this method may be adversely affected by the non-randomness of the offered traffic, intensity variation during the measurement period, or use of an incorrect value for *N*.

b) *A* is estimated from

$$A = A_H/(1 - B)$$

where *B* is the measured overflow probability. The accuracy of this method may be aversely affected by the presence of repeat bids generated by the exchange if they are included in the circuit group bid register.

It is recommended to apply both methods a) and b); any significant discrepancy would then require further investigation. It should be noted however, that both of these methods may become unreliable for high-usage groups with high overflow probability; in this situation a longer measurement period may be required for reliable results.

3.2 *High-usage group with significant congestion on the final group*

In this case, estimation of the traffic offered requires a combination of the methods of §§ 2.2 and 3.1. A proper understanding of the different parameters, through further study, is required before a detailed procedure can be recommended.

4 Origin-destination equivalent traffic offered

This section deals with the determination of equivalent traffic model according to the model described in Annex B.

An accurate estimate of origin-destination traffic offered is essential to design, engineer and service any communications network. This is especially, but not uniquely, true for dynamic routing networks.

The accuracy of this estimation depends on the availability of measurements and on the network structure.

As a matter of fact, three situations should be distinguished:

i) the origin-destination traffic offered can be directly measured by the switches of the network;

ii) the origin-destination traffic measurements are available only on a sampling basis;

iii) the origin-destination traffic measurements are not available at all.

4.1 *Determination of origin-destination traffic offered when origin-destination traffic measurements on the totality of call attempts are available*

In this case the problem of determining the origin-destination traffic offered is directly solved by the measurements as it is specified in Recommendation E.502, § 4.2.4, and no further computations are needed.

4.2 *Determination of origin-destination traffic offered when origin-destination traffic measurements only on a sampling basis are available*

These measurements should be supported by consistent measurements on the traffic volume (erlang) on the totality of outgoing traffic. More precisely, if the set of origin-destination measurements, as specified in Recommendation E.502, § 4.2.4, type 15: "traffic dispersion", is a sampling of the total traffic outgoing the exchange, the relevant measurements on traffic volume should be the overall measurements on originating outgoing traffic and on transit traffic (type 3 and type 6 respectively of the same Recommendation). If "the traffic dispersion" is performed on a specific trunk group, of course the relevant measurement on traffic volume should be performed on the same trunk group (measurement type 10). The determination of the traffic offered from measurements of the carried traffic, should be achieved by using the procedure described in § 2 of this Recommendation.

4.3 *Determination of origin-destination traffic offered when only trunk group based measurements of traffic intensity are available*

This section refers to the switches which do not perform any origin-destination measurements but only trunk group based traffic intensity measurements. The following method [1] can be applied to hierarchical and non-hierarchical networks whose routing scheme can be either fixed or updated periodically with period δT.

These two assumptions are made:

i) on each link, calls from different traffic relations see the same blocking which is the given measured trunk group blocking;

ii) if a call is routed on a 2-link path, the event that this call will be blocked on either link is independent of the event that it will be blocked on the other link.

Simulation studies have shown that these assumptions produce estimates of traffic offered for individual origin-destination pairs that are within 6 to 7% of actual values.

The following information is supposed to be available at each time interval:

i) the trunk group measurements which include the carried load and blocking on each trunk group;

ii) the (fixed) routing sequence during the δT period.

Under the above assumptions, it can be shown that the following equations holds:

$$CL = \mathrm{Z} \cdot a \qquad (1)$$

where CL is a vector whose elements are the carried traffic of each trunk group, a is a vector whose elements are the origin-destination traffic offered, and Z is a matrix whose elements are defined by the blocking on each trunk group and the routing sequence.

The origin-destination traffic offered, a, can be obtained by solving equation (1).

The notations, as well as the derivation and solution of equation (1) are described in Annex C. Two illustrative examples are given in Annex D.

ANNEX A

(to Recommendation E.501)

A simplified model for the formula presented in § 2.2

The call attempts arriving at the considered circuit group may be classified as shown in Figure A-1/E.501.

The total call attempt rate at the circuit group is

$$N = N_0 + N_{NR} + N_{LR}$$

We must consider $N_0 + N_{NR}$ which would be the call attempt rate if there were no congestion on the circuit group.

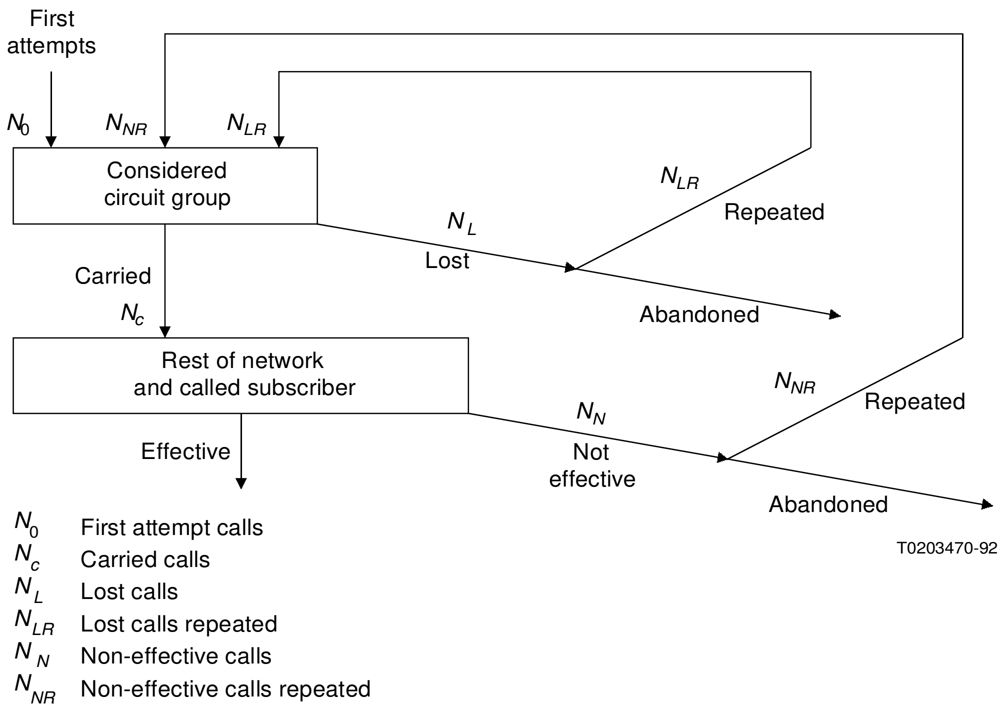

FIGURE A-1/E.501

Let

$B = \frac{N_L}{N}$ = measured blocking probability on the circuit group;

$W = \frac{N_{LR}}{N_L}$ = proportion of blocked call attempts that re-attempt.

We have

$$N_0 + N_{NR} = N - N_{LR} = (N - N_{LR})\frac{N_c}{N_c} = N_c \frac{(N - N_{LR})}{(N - N_L)} = N_c \frac{(1 - BW)}{(1 - B)}$$

Multiplying by the mean holding time of calls carried on the circuit group, h, gives

$$A = A_c \frac{(1 - BW)}{(1 - B)}$$

where

A_c is the traffic carried on the circuit group.

The above model is actually a simplification since the rate N_{NR} would be changed by augmentation of the circuit group.

An alternative procedure is to estimate an equivalent persistence W from the following formulae:

$$W = \frac{r'H}{1 - H(1 - r')}$$

$$H = \frac{\beta - 1}{\beta(1 - r)}$$

$$\beta = \frac{\text{All call attempts}}{\text{First call attempts}}$$

where r' is the completion ratio for seizures on the considered circuit group and r is the completion ratio for call attempts to the considered circuit group.

These relationships may be derived by considering the situation after augmentation (see Figure A-2/E.501).

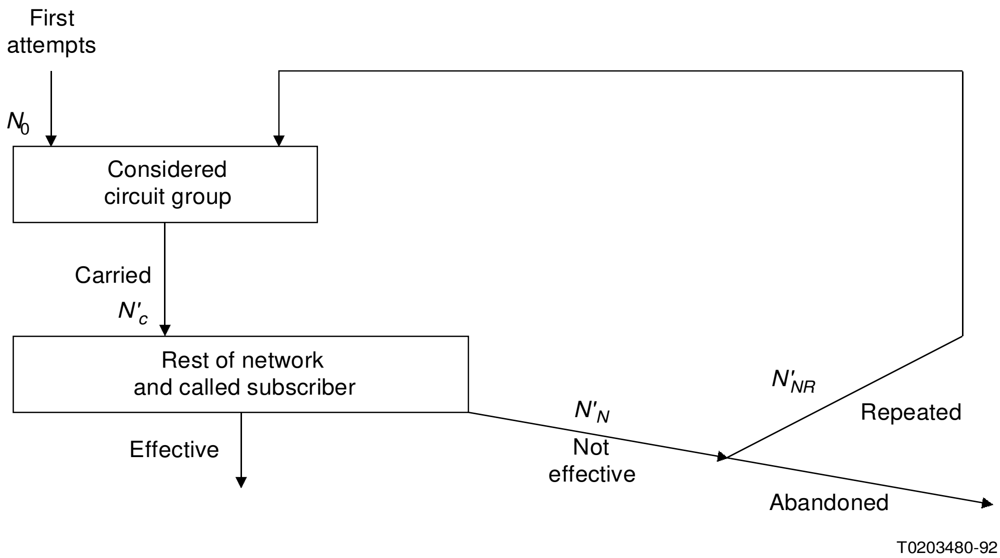

FIGURE A-2/E.501

It is required to estimate N'_c, the calls to be carried when there is no congestion on the circuit group. This may be done by establishing relationships between N_c and N_0 (before augmentation) and between N'_c, and N_0 (after augmentation), since the first attempt rate N_0 is assumed to be unchanged. We introduce the following parameters:

H is the overall subscriber persistence,

r' is the completion ratio for seizures on the circuit group.

Before augmentation:

$$H = \frac{N_{NR} + N_{LR}}{N_N + N_L}$$

$$r' = \frac{N_c - N_N}{N_c}$$

After augmentation:

$$H = \frac{N'_{NR}}{N'_N}$$

$$r' = \frac{N'_c - N'_N}{N'_c}$$

It is assumed for simplicity that H and r' are unchanged by the augmentation. The following two relationships may be readily derived:

$$N_0 = \frac{N_c\,[1 - H(1 - r') - r'BH]}{1 - B}$$

$$N_0 = N'_c\,[1 - H(1 - r')]$$

Hence

$$N'_c = \frac{N_c\left[1 - \left(\frac{r'H}{1 - H(1 - r')}\right)B\right]}{1 - B}$$

On multiplying by the mean call holding time, h, this provides our estimate of traffic offered in terms of traffic carried.

The relationship

$$H = \frac{\beta - 1}{\beta(1 - r)}$$

is valid both before and after augmentation, as may easily be derived from the above diagrams.

Note 1 – Other Administrations may be able to provide information on the call completion ratio to the considered destination country.

Note 2 – The procedure of estimating the factor W above is based on the assumptions that H, r' and h remain unchanged after augmentation. The elimination of congestion in the group considered, leads to a change in H and in practical cases, this causes an underestimation of the factor W and consequently an overestimation of offered traffic in the formula of § 2.2. A relevant study in the period 1985-88 has shown that the overestimation is practically negligible if $B \le 0.2$ and $r' \ge 0.6$. For larger B and smaller r' values, the overestimation may be significant unless other factors, not having been taken into account by the study, do not counteract. Therefore, caution is required in using Table A-1/E.501 in the indicated range. In the case of dynamically developing networks the overestimation of offered traffic and relevant overprovisioning may be tolerated, but this may not be the case for stable networks.

TABLE A-1/E.501

Values of $\frac{1 - WB}{1 - B}$

H =	0.70	0.75	0.80	0.85	0.90	0.95
B = 0.1						
r´ = 0.3	1.0653	1.0584	1.0505	1.0411	1.0300	1.0165
r´ = 0.4	1.0574	1.0505	1.0427	1.0340	1.0241	1.0129
r´ = 0.5	1.0512	1.0444	1.0370	1.0289	1.0202	1.0105
r´ = 0.6	1.0462	1.0396	1.0326	1.0252	1.0173	1.0089
r´ = 0.7	1.0421	1.0358	1.0292	1.0223	1.0152	1.0077
r´ = 0.8	1.0387	1.0326	1.0264	1.0200	1.0135	1.0068
B = 0.2						
r´ = 0.3	1.1470	1.1315	1.1136	1.0925	1.0675	1.0373
r´ = 0.4	1.1293	1.1136	1.0961	1.0765	1.0543	1.0290
r´ = 0.5	1.1153	1.1	1.0833	1.0652	1.0454	1.0238
r´ = 0.6	1.1041	1.0892	1.0735	1.0568	1.0390	1.0201
r´ = 0.7	1.0949	1.0806	1.0657	1.0503	1.0342	1.0174
r´ = 0.8	1.0872	1.0735	1.0595	1.0451	1.0304	1.0154
B = 0.3						
r´ = 0.3	1.2521	1.2255	1.1948	1.1587	1.1158	1.0639
r´ = 0.4	1.2216	1.1948	1.1648	1.1311	1.0931	1.0498
r´ = 0.5	1.1978	1.1714	1.1428	1.1118	1.0779	1.0408
r´ = 0.6	1.1785	1.1530	1.1260	1.0974	1.0669	1.0345
r´ = 0.7	1.1627	1.1382	1.1127	1.0862	1.0587	1.0299
r´ = 0.8	1.1495	1.1260	1.1020	1.0774	1.0522	1.0264
B = 0.4						
r´ = 0.3	1.3921	1.3508	1.3030	1.2469	1.1801	1.0995
r´ = 0.4	1.3448	1.3030	1.2564	1.2040	1.1449	1.0775
r´ = 0.5	1.3076	1.2666	1.2222	1.1739	1.1212	1.0634
r´ = 0.6	1.2777	1.2380	1.1960	1.1515	1.1041	1.0537
r´ = 0.7	1.2531	1.2150	1.1754	1.1342	1.0913	1.0466
r´ = 0.8	1.2325	1.1960	1.1587	1.1204	1.0813	1.0411
B = 0.5						
r´ = 0.3	1.5882	1.5263	1.4545	1.3703	1.2702	1.1492
r´ = 0.4	1.5172	1.4545	1.3846	1.3061	1.2173	1.1162
r´ = 0.5	1.4615	1.4	1.3333	1.2608	1.1818	1.0952
r´ = 0.6	1.4166	1.3571	1.2941	1.2272	1.1562	1.0806
r´ = 0.7	1.3797	1.3225	1.2631	1.2013	1.1369	1.0699
r´ = 0.8	1.3488	1.2941	1.2380	1.1807	1.1219	1.0617

ANNEX B

(to Recommendation E.501)

Equivalent traffic offered

In the lost call model the equivalent traffic offered corresponds to the traffic which produces the observed carried traffic in accordance with the relation:

$$y = A(1 - B)$$

where

y is the carried traffic,

A is the equivalent traffic offered,

B is the call congestion through the part of the network considered.

Note 1 – This is a purely mathematical concept. Physically, it is only possible to detect bids whose effect on occupancies tells whether these attempts give rise to very brief seizures or to calls.

Note 2 – The equivalent traffic offered, which is greater than the traffic carried and therefore, greater than the effective traffic, is greater than the traffic offered when the subscriber is very persistent.

Note 3 – B is evaluated on a purely mathematical basis so that it is possible to establish a direct relationship between the traffic carried and call congestion B and to dispense with the role of the equivalent traffic offered A.

ANNEX C

(to Recommendation E.501)

Methods for determination of origin-destination traffic offered when only the measurements of traffic intensity on trunk group basis are available

(Notation, derivation, and solution of equation (1) in § 4.3)

The following notations are adopted:

L : the number of links;

P : the number of traffic relations;

$a(i)$: the offered traffic for traffic relation i;

Path ij : denote the j-th route for traffic relation i;

$OL(ij)$: the traffic relation i offered to path ij;

$PB(ij)$: the path blocking of path ij;

$CL(ij)$: the traffic relation i carried on path ij;

$$CL(ij) = OL(ij) \cdot [1 - PB(ij)] \tag{C-1}$$

Path link *ijk* : denotes the *k*-th link of path *ij*:

- $k = 1$ or 2 since only 1-link and 2-link paths are considered (the extension to *n*-link path is straightforward);
- each path link *ijk* corresponds to a unique network link *q* ($q = 1, 2, ..., L$), but each network link *q* may correspond to a number of path links *ijk*. This relationship is denoted by a mapping *X*, i.e.:

  $X(ijk) = q$

q either denotes a trunk group or is equal to zero so that:

- if $X(ij1) = 0$, it means that traffic relation *i* has at most *j*-1 routes;
- if $X(ij2) = 0$ and $X(ij1) \neq 0$, it means that *j*-th route for traffic relation *i* is a 1-link path;

LB(*ijk*): the link blocking of link *ijk*;

CL(*q*): the total traffic carried on link *q*.

Because of the assumptions on the independence of the call blocking on each link of a path, the path blocking is a simple function of its link blockings:

$$PB(ij) = LB(ij1) + LB(ij2) - LB(ij1) \cdot LB(ij2)$$

When there is the crankback capability, the following equation can be derived:

$$OL(ij) = a(i) \cdot \prod_{t=1}^{j-1} PB(it) \qquad \text{(C-2)}$$

Therefore, from equations (C-1) and (C-2):

$$CL(ij) = a(i) \cdot [1 - PB(ij)] \cdot \prod_{t=1}^{j-1} PB(it) = s(ij) \cdot a(i)$$

where

$$s(ij) = [1 - PB(ij)] \cdot \prod_{t=1}^{j-1} PB(it)$$

Then the total carried traffic on each link *q* is:

$$CL(q) = \sum_{X(ijk)=q} CL(ij) = \sum_{X(ijk)=q} s(ij) \cdot a(i) \qquad \text{(C-3)}$$

When there is no crankback capability, a call will be routed in the next route in the routing sequence only if it is blocked on the first link of path *ij*. The call will be abandoned if it is blocked on the second link. In this case the equation (C-2) must be rewritten in the following way:

$$OL(ij) = a(i) \cdot \prod_{t=1}^{j-1} LB(it1) \qquad \text{(C-2}')$$

From equations (C-1) and (C-2′):

$$CL(ij) = a(i) \cdot [1 - PB(ij)] \cdot \prod_{t=1}^{j-1} LB(it1)$$

and assuming in this case (no crankback capability):

$$s(ij) = [1 - PB(ij)] \cdot \prod_{t=1}^{j-1} LB(it1)$$

the final equation can be written as in (C-3).

The offered traffic for each relation can be derived from the set of equations (C-3), in which the definition of $s(ij)$ is depending on the presence of crankback capability on the network.

Pseudo-inverse and various iterative methods can be applied to solve the equation system (C-3), that can be written in the following matrix form (see Note):

$$CL = Z \cdot a \qquad \text{(C-4)}$$

where

$$CL = [CL(1) \ldots\ldots\ldots\ldots CL(L)]^T$$
$$a = [a(1) \ldots\ldots\ldots\ldots a(P)]^T$$
$$Z = [z(u,v)]$$

where $z(u,v) = s(v,r)$ if link u is a link of the r-th path of relation v; otherwise is equal to zero.

Iterative methods are usually most efficient for solving the equations resulting from a large network. However, if the pseudo-inverse method is used, the solution of the system (C-4) is:

$$a^\circ = Z^\circ \cdot CL$$

where a° is the estimated offered traffic relation and Z° is the pseudo-inverse of Z [2].

If the system is square, namely the number of equations is equal to the number of unknowns and therefore, the network is fully meshed, the solution is univocally determined, in fact:

$$Z^\circ = Z^{-1}$$

For non-fully connected networks, namely the number of equations is less than the number of unknowns, the equation system does not have a unique solution, therefore the offered traffic for each relation must be estimated, introducing in this way an error that is greater as the number of trunk groups decreases. In this case:

$$Z^\circ = Z^T \cdot (Z \cdot Z^T)^{-1}$$

Finally, there can also be the case in which the number of equations is greater than the number of unknowns (overdetermined systems). This can happen, for example, when other network measurements, such as the office totals, are added. In this case:

$$Z^\circ = (Z^T \cdot Z)^{-1} \cdot Z^T$$

In any case, $a°$ is the optimal estimate of a, in the least-square sense, based on the available measurements.

Note 1 – The symbols T and -1 in the vector and matrix equations represent the mathematical operations *transpose* and *inverse* respectively.

Note 2 – Different traffic relations do not see the same blocking on a link, especially if circuit reservation is used. The proposed method can make use of these different values if they are available. In this case, $LB(ijk)$ should be interpreted as the link blocking seen by the traffic relation i on the link ijk. The derived equations remain unchanged.

ANNEX D

(to Recommendation E.501)

Examples using method described in Annex C

Example 1

Consider the following 3 node network:

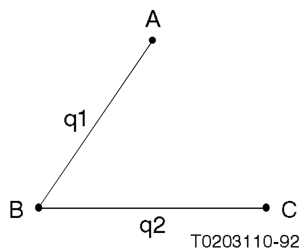

FIGURE D-1/E.501

The point pairs and their only routes are given in the following table:

Point-pair	(A, B)	(B, C)	(C, A)	(B, A)	(C, B)
i	1	2	3	4	5
Only route	$q1$	$q2$	$q2$, $q1$	$q1$	$q2$

Based on the routing table, the mapping $X(ijk) = q$ can be expressed as follows:

ijk	*q*	*ijk*	*q*	*ijk*	*q*	*ijk*	*q*	*ijk*	*q*	*ijk*
111	1	211	2	311	2	411	1	511	2	611
112	0	212	0	312	1	412	0	512	0	612

The Z matrix is:

$$Z = \begin{bmatrix} s(11) & 0 & s(31) & s(41) & 0 & s(61) \\ 0 & s(21) & s(31) & 0 & s(51) & s(61) \end{bmatrix}$$

Let us assume that all the links have the same blocking value of 0.1, then, we obtain the values for $s(ij)$:

$s(i1) = 0.9$ for $i = 1, 2, 4$ and 5, and $s(i1) = 0.81$ for $i = 3$ and 6.

Thus we have:

$$\begin{bmatrix} CL(1) \\ \\ CL(2) \end{bmatrix} = \begin{bmatrix} 0.9 & 0 & 0.81 \\ \\ 0 & 0.9 & 0.81 \end{bmatrix} \begin{bmatrix} a1 + a4 \\ a2 + a5 \\ a3 + a6 \end{bmatrix}$$

Assuming $CL(1) = 5$ erl, and $CL(2) = 7$ erl, we obtain:

$$\begin{bmatrix} a1 + a4 \\ a2 + a5 \\ a3 + a6 \end{bmatrix} = \begin{bmatrix} 1.43 \text{ erl} \\ 3.65 \text{ erl} \\ 4.58 \text{ erl} \end{bmatrix}$$

That is, the (two way) offered traffic between point-pair (A, B) is 1.43 erl, between point-pair (B, C) is 3.65 erl, and between point-pair (A, C) 4.58 erl.

Example 2

Consider the following network:

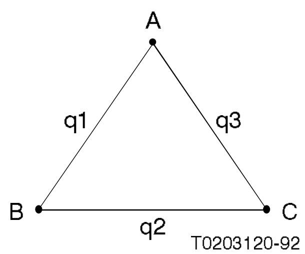

FIGURE D-2/E.501

The point-pairs and their routing sequences are given in the following tables.

Point-pair	(A, B)	(B, C)	(C, A)	(B, A)	(C, B)	(A
i	1	2	3	4	5	
1st choice route	$q1$	$q2$	$q3$	$q1$	$q2$	[illegible]
2nd choice route	$q3, q2$	$q1, q3$	$q2, q1$	$q2, q3$	$q3, q1$	$q1$

Based on the routing table, the mapping $X(ijk) = q$ can be expressed as follows:

ijk	*q*	*ijk*	*q*	*ijk*	*q*	*ijk*	*q*	*ijk*	*q*
111	1	211	2	311	3	411	1	511	2
112	0	212	0	312	0	412	0	512	0
121	3	221	1	321	2	421	2	521	3
122	2	222	3	322	1	422	3	522	1

The Z matrix is:

$$Z = \begin{bmatrix} s(11) & s(22) & s(32) & s(41) \\ s(12) & s(21) & s(32) & s(42) \\ s(12) & s(22) & s(31) & s(42) \end{bmatrix}$$

Let us assume that all the links have the same blocking value of 0.1. Then, we obtain the following values for $s(ij)$ for both with and without crankback: $s(i1) = 0.9$ and $s(i2) = 0.81$.

Thus we have:

$$\begin{bmatrix} CL(1) \\ CL(2) \\ CL(3) \end{bmatrix} = \begin{bmatrix} 0.9 & 0.081 & 0.081 \\ 0.081 & 0.9 & 0.081 \\ 0.081 & 0.081 & 0.9 \end{bmatrix} \begin{bmatrix} a1 + a4 \\ a2 + a5 \\ a3 + a6 \end{bmatrix}$$

Assuming $CL(1) = 5$ erl, $CL(2) = 7$ erl, and $CL(3) = 10$ erl, we obtain:

$$\begin{bmatrix} a1 + a4 \\ a2 + a5 \\ a3 + a6 \end{bmatrix} = \begin{bmatrix} 4.06 \text{ erl} \\ 6.05 \text{ erl} \\ 10.16 \text{ erl} \end{bmatrix}$$

That is, the (two way) offered traffic between point-pair (A, B) is 4.06 erl, between point-pair (B, C) 6.05 erl, and between point-pair (A, C) 10.61 erl.

References

[1] TU (M.): *Estimation of origin-destination traffic demand in dynamic routing networks*. Proc. 13th ITC, Copenhagen, 1991.

[2] ALBERT (A): *Regression and the Moore-Penrose Pseudoinverse*. Academic Press, New York, 1972.