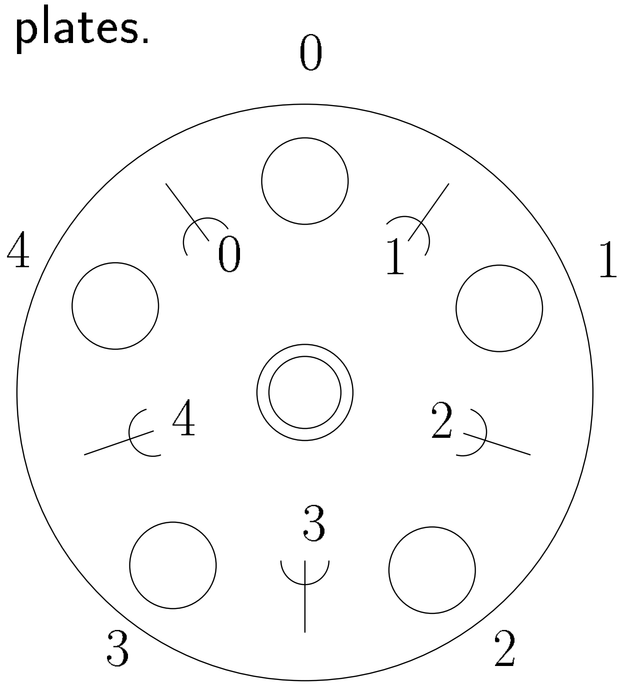
◇ The Dining Philosophers ◇

Five philosophers live in a college; they spend most of their time thinking, but occasionally become hungry. The college has a communal dining room, with a circular table and five chairs. In the middle of the table is a large bowl of spaghetti, and the table is set with five plates. There are also five forks, one between each pair of plates.



When a philosopher is hungry, he enters the dining room, sits down in his chair, picks up the forks on either side of his plate (first the fork on the left, then the fork on the right), and eats. Two forks are needed to eat spaghetti, so if one of the forks is already in use, he has to wait. When the philosopher has finished eating he puts down the forks, gets up from the chair, and leaves the dining room.

We will model this system in CSP, and analyse its behaviour. The relevant components are the five philosophers, which we will model as processes $PHIL_0 \ldots PHIL_4$, and the five forks, which we will model as processes $FORK_0 \ldots FORK_4$.

Using the symbols $\oplus$ and $\ominus$ to denote addition and subtraction modulo 5 (so that $4 \oplus 1 = 0$ and $0 \ominus 1 = 4$), philosopher $PHIL_i$ will sit in seat i and use forks i and $i \oplus 1$.

The alphabet of $PHIL_i$ is

$$\begin{aligned} \alpha PHIL_i \;=\; \{ & i.\mathit{sitdown}, i.\mathit{getup}, \\ & i.\mathit{pickup}.i, i.\mathit{pickup}.(i \oplus 1), \\ & i.\mathit{putdown}.i, i.\mathit{putdown}.(i \oplus 1)\} \end{aligned}$$

In the events $i.\mathit{pickup}.i$ etc. the "." is being used purely as a symbol.

The event $i.\mathit{pickup}.i$ represents $PHIL_i$ picking up fork i, and so on.

We will ignore the actions of eating, thinking, and entering and leaving the dining room.

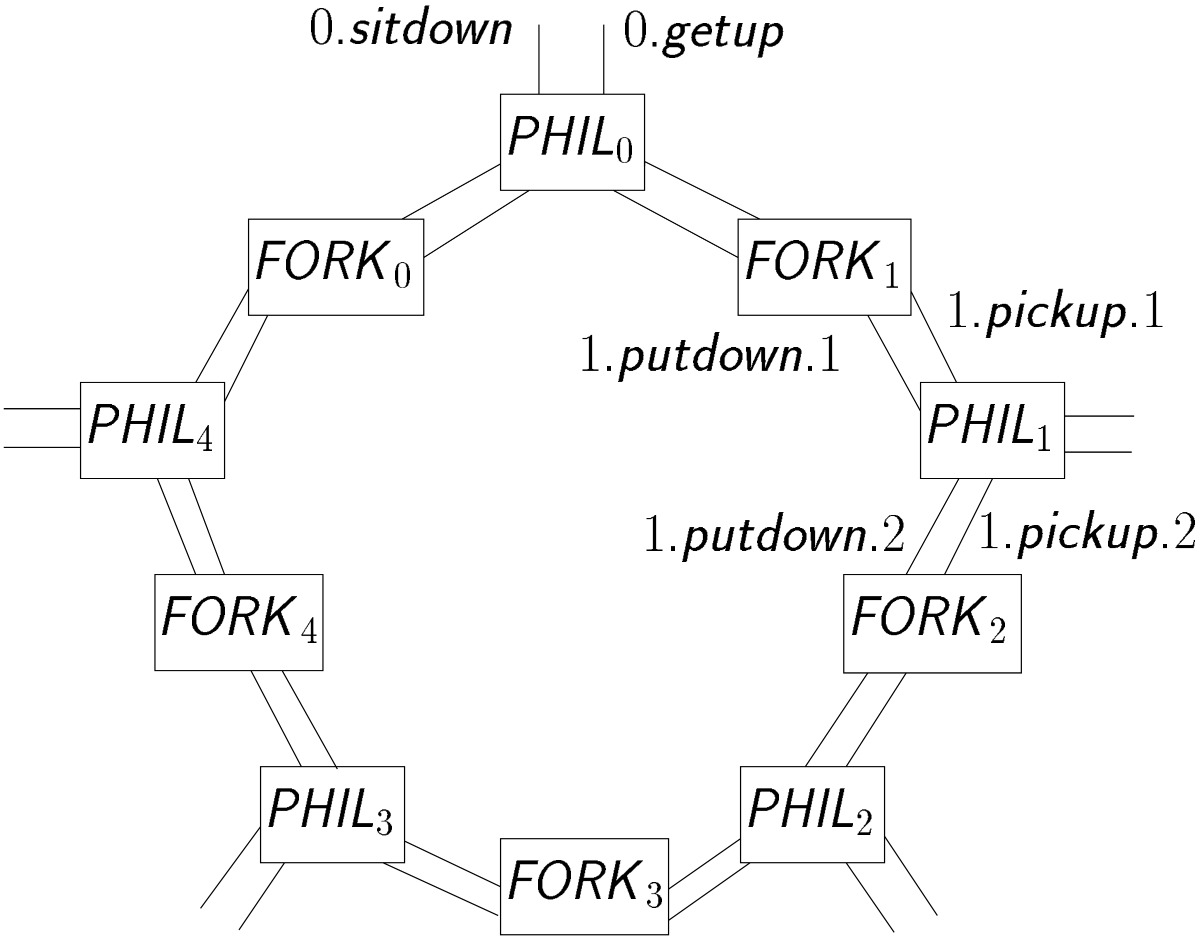
Because the alphabets of the processes $PHIL_i$ are mutually disjoint, there can be no direct interaction between the philosophers. The only way in which they affect each other will be as a consequence of the fact that they are competing for access to the forks.

The relevant events for the forks are the *pickup* and *putdown* events. FORK_i can potentially be picked up or put down by either PHIL_i or $\mathit{PHIL}_{i\ominus 1}$.

$$\begin{aligned}\alpha\mathit{FORK}_i \;=\; & \{i.\mathit{pickup}.i, (i\ominus 1).\mathit{pickup}.i,\\ & i.\mathit{putdown}.i, (i\ominus 1).\mathit{putdown}.i\}\end{aligned}$$

We will define the system as a concurrent combination of the philosophers and the forks.

$$\begin{aligned}\mathit{PHILS} &= \mathit{PHIL}_0 \parallel \mathit{PHIL}_1 \parallel \mathit{PHIL}_2 \parallel \mathit{PHIL}_3 \parallel \mathit{PHIL}_4\\ \mathit{FORKS} &= \mathit{FORK}_0 \parallel \mathit{FORK}_1 \parallel \mathit{FORK}_2\\ &\quad \parallel \mathit{FORK}_3 \parallel \mathit{FORK}_4\\ \mathit{COLLEGE} &= \mathit{PHILS} \parallel \mathit{FORKS}\end{aligned}$$



Each process $PHIL_i$ simply cycles through a sequence of six events:

$$\begin{aligned} PHIL_i \;=\; & i.\mathit{sitdown} \rightarrow i.\mathit{pickup}.(i \oplus 1) \rightarrow i.\mathit{pickup}.i \rightarrow \\ & i.\mathit{putdown}.(i \oplus 1) \rightarrow i.\mathit{putdown}.i \rightarrow \\ & i.\mathit{getup} \rightarrow PHIL_i \end{aligned}$$

Each process $FORK_i$ can be repeatedly picked up and put down, but there is a choice of who picks it up:

$$\begin{aligned} & FORK_i = \\ & \quad i.\mathit{pickup}.i \rightarrow i.\mathit{putdown}.i \rightarrow FORK_i \\ & \quad \Box\ (i \ominus 1).\mathit{pickup}.i \rightarrow (i \ominus 1).\mathit{putdown}.i \rightarrow FORK_i \end{aligned}$$

Now we can look at the possible behaviour of $COLLEGE$. Suppose all the philosophers sit down in order, and then each one picks up the fork to his left.

What can happen next? Each $PHIL_i$ can only do $i.\mathit{pickup}.i$, which requires synchronisation with $FORK_i$. However, $FORK_i$ has just done $(i \ominus 1).\mathit{pickup}.i$ and therefore can only do $(i \ominus 1).\mathit{putdown}.i$ next. This means that there is no possible next event for $COLLEGE$. We have a deadlock.

How can we modify *COLLEGE* to remove the possibility of deadlock? There are a number of obvious but unsatisfactory ideas.

- ◇ Provide two forks for each philosopher. But if the forks represent scarce resources, this may not be feasible.
- ◇ Provide a single extra fork, in the middle of the table, which can be used by any of the philosophers. Similarly, this may not be feasible.
- ◇ Modify the definition of just one of the philosophers, so that the forks are picked up in the opposite order. This will work (although it takes some thought to be sure) but it breaks the symmetry of the system.

Instead we will try to control the way in which the philosophers sit down, the idea being that if only 4 philosophers are seated at any one time, then even if everyone picks up the left fork, one philospher will be sitting on the left of an empty place, and can pick up the fork to his right.

As we have seen, the behaviour of a system can be controlled by adding another process in parallel, and taking advantage of the fact that certain events require synchronisation.

We can define a process *BUTLER* with alphabet

$$\alpha \mathit{BUTLER} = D \cup U,$$

where

$$\begin{aligned} D &= \{0.\mathit{sitdown}, \ldots, 4.\mathit{sitdown}\} \\ U &= \{0.\mathit{getup}, \ldots, 4.\mathit{getup}\}, \end{aligned}$$

to control the sitting down and getting up of the philosophers. *BUTLER* is defined in terms of auxiliary processes $\mathit{BUTLER}_0, \ldots, \mathit{BUTLER}_4$, all with alphabet $\alpha \mathit{BUTLER}$.

$$\begin{aligned} \mathit{BUTLER}_0 &= x : D \to \mathit{BUTLER}_1 \\ \mathit{BUTLER}_i &= x : D \to \mathit{BUTLER}_{i+1} \\ &\;\Box\; y : U \to \mathit{BUTLER}_{i-1} \quad 1 \leqslant i \leqslant 3 \\ \mathit{BUTLER}_4 &= y : U \to \mathit{BUTLER}_3 \\ \mathit{BUTLER} &= \mathit{BUTLER}_0 \end{aligned}$$

The notation in the second line is shorthand for

$$\mathit{BUTLER}_i = z : (D \cup U) \to P(z)$$

where

$$\begin{aligned} P(z) &= \mathit{BUTLER}_{i+1} \text{ if } z \in D \\ &= \mathit{BUTLER}_{i-1} \text{ if } z \in U. \end{aligned}$$

Now we can define

$$\mathit{NEWCOLLEGE} = \mathit{COLLEGE} \parallel \mathit{BUTLER}$$

△ Convince yourself that *NEWCOLLEGE* does not deadlock. How formal can you be?

We could consider checking the entire state space of the system, to discover whether or not it can deadlock. Since each philosopher has 6 states and each fork has 3 states, the total number of possible states of *COLLEGE* is $6^5 \times 3^5$, or about 1.8 million. Since the effect of *BUTLER* is to restrict the number of states, this is also a limit on the number of states of *NEWCOLLEGE*. Systems of this complexity are within the scope of current software tools such as FDR.