Notes on Model of Diode Laser Spectra

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According to the *Wiener-Khinchine Theorem*, the optical spectrum is given by the Fourier transform of the field autocorrelation function [1]

$$S_E(\omega) = \int_{-\infty}^{+\infty} \Phi(\tau) e^{-i\omega\tau} d\tau \tag{1}$$

The field autocorrelation function $\Phi(\tau)$ is given by

$$\Phi(\tau) = \langle E^*(t+\tau)E(t) \rangle \tag{2}$$

where the brackets denote averaging over t. The time dependent E field of a laser can be expressed as

$$E(t) = \sqrt{P + \delta P(t)} \cdot e^{-i(\omega_0 t + \phi + \delta\phi(t))} \tag{3}$$

where P and ϕ are the steady state power and initial phase values, and $\delta P(t)$ and $\delta\phi(t)$ represent the random power and phase fluctuations inside the laser. In diode lasers spontaneous emission is the source of $\delta P(t)$ and $\delta\phi(t)$. Given suitable random distributions for $\delta P(t)$ and $\delta\phi(t)$, the above equations may be integrated numerically to evaluate the optical spectrum $S_E(\omega)$. In the simplest approximation $\delta P(t)$ is taken to be zero, and thus

$$\Phi(\tau) \approx P e^{i\omega_0\tau} \left\langle e^{i(\delta\phi(t+\tau) - \delta\phi(t))} \right\rangle \tag{4}$$

The following notation will be used from hereon to simplify expressions:

$$\Delta\phi(t,\tau) = \delta\phi(t+\tau) - \delta\phi(t) \tag{5}$$

Since spontaneous emission events are random, we can assume $\Delta\phi(t,\tau)$ to be a Gaussian random variable. Then it can be shown by using the result for the mean of the function of a random variable X,

$$\langle f(X)\rangle = \int_{-\infty}^{+\infty} f(x)p(x)dx \tag{6}$$

where $p(x)$ is the probability density function for X, that

$$\left\langle e^{i\Delta\phi(t,\tau)}\right\rangle = e^{-\frac{1}{2}\langle(\Delta\phi(t,\tau))^2\rangle} \tag{7}$$

and the time average $\langle(\Delta\phi(t,\tau))^2\rangle$ can be evaluated from a transient analysis of the laser rate equations to a sudden change in the field intensity caused by the spontaneous emission event. This yields the following expression in terms of phenomenological parameters [1][2]:

$$\langle(\Delta\phi(t,\tau))^2\rangle = \frac{R_{sp}}{2P}\left((1+\alpha^2 b)\tau + \frac{\alpha^2 b}{2\Gamma_R\cos(\delta)}[\cos(3\delta) - e^{-\Gamma_R\tau}\cos(\Omega_R\tau - 3\delta)]\right) \tag{8}$$

where

$$b = \frac{\Omega_R}{\sqrt{\Omega_R^2 + \Gamma_R^2}} \tag{9}$$

and

$$\delta = \tan^{-1}\left(\frac{\Gamma_R}{\Omega_R}\right) \tag{10}$$

$\Omega_R = 2\pi\nu_R$, where ν_R is the relaxation oscillation frequency, Γ_R is the damping rate of the relaxation oscillations, α is the linewidth enhancement factor, and R_{sp} is the spontaneous emission rate. Putting the expression for $\langle(\Delta\phi(t,\tau))^2\rangle$ from equation 8 into equation 7, and substituting the resulting expression into equation 4, the field autocorrelation function in the approximation $\delta P(t) = 0$ is given by

$$\Phi(\tau) \approx Pe^{i\omega_0\tau}e^{-\frac{R_{sp}}{4P}\left((1+\alpha^2 b)\tau + \frac{\alpha^2 b}{2\Gamma_R\cos(\delta)}[\cos(3\delta) - e^{-\Gamma_R\tau}\cos(\Omega_R\tau - 3\delta)]\right)} \tag{11}$$

Equation 11 describes a *real* autocorrelation function; hence, its Fourier transform will yield a *symmetric* spectrum. The spectrum is centered at frequency ω_0. Its central peak has a Lorentzian profile and symmetric sidebands are present at $\omega = \pm\Omega_R$. Real diode laser spectra are asymmetric. An analysis of the spectrum including correlated power fluctuations was developed by van Exter, *et. al.* [3].

References

[1] G. P. Agrawal and N. K. Dutta, *Semiconductor Lasers*, New York: Van Nostrand Reinhold, 1993.

[2] C. H. Henry, *Theory of the phase noise and power spectrum of a single mode injection laser*, IEEE J. Quant. Electron., **QE-19**, 1391.

[3] M. P. van Exeter, W. A. Hamel, J. P. Woerdman, and B. R. P. Zeijlmans, *Spectral signature of relaxation oscillations in semiconductor lasers*, IEEE J. Quant. Electron., **28**, 1470.