Notes for Seminar on Bell's Theorem

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1 Background

1.1 Particles of Light and Waves of Matter

Some experimental observations:

1. A very low intensity beam of light reveals itself to consist of discrete particles (*photons*), as evidenced by individual electrical pulses from the output of a photomultiplier.

2. Electrons passing through a double slit produce an interference pattern on a screen. Even if one electron passes through at a time, the pattern is accumulated over time.

1.2 Physical Reality of Observable Properties

Observable physical properties (of photons, electrons, ...) include:

position

momentum

energy

angular momentum

spin

Common sense expects that a particle will have *definite values* of all of these properties at any instant in time. Classical physics (prior to quantum mechanics) tacitly assumed that disturbances arising from the measurement process can be made arbitrarily small. Einstein, Podolsky, and Rosen (1935) stated the following criterion for an observable property to be considered physically real:

> If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity.

Quantum mechanics, on the other hand, predicts that *certain pairs of properties*, such as position and momentum, or any two orthogonal components of angular momentum, cannot be known exactly *at the same time.* For these special pairs of properties, the measurement of one (momentum) will disturb the second (position), and vice-versa. The crucial prediction of the theory is that *the disturbance cannot be made arbitrarily small.* Furthermore, the disturbance acts such that the smaller we try to make the uncertainty in one property (position), the greater we make the uncertainty in the second property (momentum).

An example of this behavior is given by the result of electrons passing through a narrow slit. The narrower the slit, the smaller the uncertainty in position of each electron. However, upon emerging from the slit, the electrons are found to be deflected over a wider range of angles indicating an increase in the uncertainty in momentum.

Therefore, we can never predict *with certainty* the *simultaneous* values of momentum and position (or certain other pairs of properties) for a particle. By the EPR criterion, such pairs of properties cannot have *simultaneous reality.*

1.3 Postulates of Quantum Mechanics

1. A wave function Ψ describes the *state* of a *system.* Ψ is a *complex* function of as many variables as required to describe the system, and Ψ obeys the Schroedinger Equation. Ψ determines *the probabilities of the possible outcomes* of any measurement made on the system.

2. The possible outcomes of the measurement of an observable property are the *eigenvalues* λ_i of an operator A associated with that observable.

3. After the measurement of an observable property (represented by operator A) is completed, the wave function *reduces* or *collapses* to the eigenfunction ϕ_n corresponding to the measured value λ_n.

These are the fundamental postulates. In the first postulate, a "system" can refer to a single electron, a multi-electron atom, the nucleus of an atom, a solid, *etc.* The Schroedinger equation is often written as

$$H\Psi = i\hbar\frac{\partial\Psi}{\partial t} \tag{1}$$

where H is the energy operator, also called the *Hamiltonian* operator. The Hamiltonian operator describes all of the forces acting upon the system. Operators are explained below.

The second postulate indicates that an operator may be found to correspond to each observable property. An operator is simply a recipe or algorithm for transforming one function into another. For example, the differential operator d/dx operating on a function Ψ simply takes the derivative of Ψ with respect to x and produces the new function Ψ'. Special functions called the *eigenfunctions* of A have the following property

$$A\phi_i = \lambda_i\phi_i \tag{2}$$

Operating on the function ϕ_i with A returns ϕ_i multiplied by the constant λ_i. The constant is called an *eigenvalue*. The operator A may have a discrete set of eigenvalues, a continuous range of eigenvalues, or a combination of both. Certain restrictions are placed on operators if they are to correspond to physically observable properties: the operators must be *linear* and they must have *real* eigenvalues.

The linear property of an operator A, plus additional *completeness* and *orthogonality* conditions on the eigenfunctions of A, allows the measurement probability of the various eigenvalues of A to be calculated for a system in some arbitrary initial state Ψ_0. The wave function can be written as a superposition (or linear combination) of the eigenfunctions of A:

$$\Psi_0 = \Sigma c_i\phi_i \tag{3}$$

The probability of measuring λ_n for the observable A is given by the magnitude squared of the expansion coefficient:

$$P = c_n^* c_n \tag{4}$$

The third postulate states that after the measurement A is performed, the state of the system is

$$\Psi = \phi_n \tag{5}$$

Subsequent measurements of A for the system will always give the same result, namely λ_n. Thus we can *predict with certainty* ($P = 1$) the outcome of measuring A once again.

If, having measured A, we subsequently measure another property B which is not compatible with A, by which we mean that the measurement of B disturbs the value of A, the state of the system after measuring B will no longer be an eigenstate of A. The outcome of measuring A will once again be statistical instead of being certain. Such a situation arises if

$$AB\Psi \neq BA\Psi \tag{6}$$

i.e., the operators A and B do not *commute*. Momentum and position operators do not commute.

1.4 The Copenhagen Interpretation of Quantum Mechanics

Neils Bohr is largely responsible for the interpretation of the postulates and predictions of quantum mechanics, widely referred to as the Copenhagen interpretation. The Copenhagen interpretation is the prevalent doctrine for contemporary physicists. This doctrine maintains that the wavefunction Ψ contains the maximum information that can be known about a system. Prior to the measurement of some property, given Ψ, we can calculate the probabilities for a set of possible outcomes of the measurement; however, we cannot predict the outcome with certainty unless Ψ happens to be an eigenstate of the measurement operator.

The Copenhagen interpretation asserts that not only is it meaningless to ask whether the system had a definite value of some property prior to its measurement, but that the property has no physical reality prior to the measurement. Bohr argues that the outcome of a measurement on a system cannot be independent of the entire experimental configuration — the measurement apparatus interacts with the system being measured (in some unspecified and inherently unknowable way), leading to the collapse of the wave function and producing a definite value for the measured property. The wave function collapse cannot be further explained. However, the act of measurement does bring about a state for which we can predict with certainty, and without disturbing the system, the value of that property, in essence providing it with physical reality consistent with the EPR definition.

1.5 A Peek at Hidden Variables and EPR

Einstein and others, later including John Bell, did not accept the idea that a system did not have definite values of observable properties prior to measurement. They were bothered by the fuzzy explanation of the collapse of the wavefunction, and the assertion that it was an irreducible phenomenon, *i.e.* could not be explained on a more fundamental basis. Obviously the statistical predictions of quantum mechanics could not be refuted since they were borne out by measurements. One way around the dilemna was to assume that quantum mechanics was incomplete and that the system possessed *hidden variables* to account for the statistical outcomes of a measurement, but that the physically observable properties of a system all had definite values. In 1935 Einstein, Podolsky, and Rosen contrived a gedanken experiment to show that quantum mechanics was not complete. For a system consisting of two *correlated particles*, they showed that either the momentum or position of the second particle can be known with certainty from a corresponding measurement performed on the first particle. The second particle may be removed by a vast distance from the first, and therefore cannot be disturbed by a measurement performed on the first particle. Since position or momentum could be measured for the first particle, at the experimenter's discretion, they argued that the second particle must have had definite values of both properties all along. The only alternative would be for the measurement of the first particle's position or momentum to instantly affect the corresponding property of the second particle, across a vast distance. Addressing this alternative, they write

> This makes the reality of P [momentum] or Q [position] depend upon the process of measurement carried out on the first system, which does not disturb the second system in any way. No reasonable definition of reality could be expected to permit this.