PHY 689-01, Spring 2009
Laser Spectroscopy of Atomic Systems
Final Exam

27 April 2009

Answer questions 1 and 2. Choose and answer 2 more questions from 3–5.

1. A *classical* dipole oscillator is driven near resonance by a monochromatic plane wave. The detuning from resonance is Δ, and the damping rate is γ.

   a) Write the expressions, in the long-time limit ($t \gg \frac{1}{\gamma}$), for the oscillator amplitude, $A(t, \Delta)$, and oscillator phase shift, $\phi(t, \Delta)$.

   b) Show that the phase shift goes to zero at large detunings.

   c) Relative to on-resonance ($\Delta = 0$), by what factor is the amplitude, A, decreased at a detuning of $\Delta = \gamma$?

   d) How does the absorption coefficient, α, scale with the oscillator amplitude, A, and, therefore, how much of a decrease in the absorption coefficient is expected at a detuning of γ, relative to α_0?

2. A constant intensity monochromatic beam excites a two-level atom on resonance, at a wavelength of 894 nm. The beam intensity is 10 mW/cm^2. The lifetime of the upper level is 34.9 ns. In the *steady-state*,

   a) What is the rate of excitation of the atom?

   b) What is the rate of stimulated emission?

   c) What is the value of the saturation parameter, S, at the given intensity?

   d) What are the populations for the upper and lower levels of the atom?

   e) What is the ratio of the on-resonant absorption coefficient, α, at the given intensity, to α_0, the on-resonant absorption coefficient in the limit of zero intensity?

   f) What is the minimum width of the absorption line in the limit of zero longitudinal velocity? How does this width differ from the width predicted by the classical model of question 1?

3. The time-averaged energy density of an abritrary electromagnetic field in free space is given by

$$\rho = \frac{1}{2}\langle\ \vec{E}\cdot\vec{D} + \vec{B}\cdot\vec{H}\ \rangle_t$$

where $\langle\ \rangle_t$ denotes time-averaging.

   a) For a monochromatic plane wave, show that the time-averaged intensity is related to the electric field amplitude by,

   $$I = \frac{1}{2}c\epsilon_0 {E_0}^2$$

   b) For the beam in question 2, assume a plane wave. What is the electric field amplitude, E_0?

   c) For the beam in question 2, assume a Lorentzian spectrum, with a full width at half maximum of 1 MHz. What is the peak spectral energy density, $\rho(\nu_0)$?

4. The expectation value of the electric dipole moment of an atom is defined by

$$\langle\ \vec{d}\ \rangle \equiv \langle\ \Psi \mid \vec{d} \mid \Psi\ \rangle$$

where $\vec{d}$ is the electric dipole moment operator. For a two-level, one-electron atom,

$$|\Psi\rangle = c_1|\lambda_1\rangle + c_2|\lambda_2\rangle$$

and $\vec{d} = -e\vec{r}$.

   a) Assume a spherically symmetric potential for the atom, and show that $\langle\ \vec{d}\ \rangle$ is given by

   $$\langle\ \vec{d}\ \rangle = 2 \times Re\{c_1^* c_2 \langle\lambda_1|\ \vec{d}\ |\lambda_2\rangle\}$$

   b) Write down the solutions for $c_1(t)$ and $c_2(t)$ when the atom is driven by a resonant, monochromatic optical field. Ignore spontaneous emission decay, and use any convenient initial conditions.

   c) Using the solutions from b), determine the magnitude of the time-dependent dipole moment, $|\langle\ \vec{d}(t)\ \rangle|$. What frequencies are present in $|\langle\ \vec{d}(t)\ \rangle|$?

5. A two-level atom, initially in the ground state, is subjected to a resonant pulse of light.

   a) For a $\pi/2$ pulse, write down the resulting state vector for the atom.

   b) What is the probability of finding the atom in the upper level?

   c) How would the results for a) and b) differ if we were to use a $3\pi/2$ pulse instead?

   d) Draw the Bloch vector for the initial state of the atom, and the final state of the atom following the $\pi/2$ pulse.