

Spreadsheet template that demonstrates a simple application of Markowitz Portfolio Optimization Strategy.

This spreadsheet uses past returns as the measure of the expected return and the past variation in return (the variance) as a measure of risk. The relationships between investments, the tendency they have to move together or opposite each other, is measured by an estimate of the covariance between them.

The idea is that for any given set of investments and a desired return there is a specific mix that minimizes the risk associated with that return. Once you know the minimum-risk-vs-return curve you can select the risk/return trade-off that suits your own investment style.

This spreadsheet sets up the basic formulas to permit hand or program optimization of a portfolio's contents to minimize risk for a given desired return or maximize return for a given allowed risk.

This material is based on a LOTUS magazine article published c. 1991.

- - - - -
1. Compute Average Return, Variance, and Covariance of the investments being considered.

The first step is to acquire the total return data for the set of investments being considered. That information is usually available from the company that issues the investment (e.g. mutual funds)

	----- total return data -----		
	Invest1	Invest2	Invest3
Year 1	20.00%	8.00%	8.00%
Year 2	10.00%	15.00%	10.00%
Year 3	4.00%	19.00%	8.00%
Year 4	26.00%	8.00%	6.00%
Year 5	20.00%	10.00%	8.00%
Avg	16.00%	12.00%	8.00%
Var	0.6240%	0.1880%	0.0160%

Note: Using more than 5 years is just fine. Just stretch the spreadsheet accordingly. But pick a period for which all (or most) of the investments have data available.

2. Estimates of co-variance between every pair of investments

	I1*I2	I1*I3	I2*I3
	1.60%	1.60%	0.64%
	1.50%	1.00%	1.50%
	0.76%	0.32%	1.52%
	2.08%	1.56%	0.48%
	2.00%	1.60%	0.80%
	1.5880%	1.2160%	0.9880%
CoVar	-0.3320%	-0.0640%	0.0280%

NOTE: that some versions of 1-2-3 handle blank entries from the above formula incorrectly and average them in as a 0. That will result in an incorrect average in the covariance. To avoid that erase any "delete me" entries that arise from one or more missing pieces of total return data.

Negative covariance means that the investments tend to move opposite each other. When one does well, the other tends not to.

----- data summary -----

	Return	Variance
Invest1	16.000%	0.624%
Invest2	12.000%	0.188%
Invest3	8.000%	0.016%
Covariance I1xI2		-0.0033200
Covariance I1xI3		-0.0006400
Covariance I2xI3		0.0002800

- - - - -

3. Optimize the individual investment amounts to acheive financial goals.

You can enter various investment mixes directly in the following table,

noting the expected return and minimum probable return (a measure of the net risk). Then find a mix that suits your style. As the number of investments grows large this quickly becomes unwieldy and one must resort to computational methods, such as an Add-In optimizer.

This table uses the data entered in the Data Summary Table under item 2.

	Fraction	Fraction	Variance calculation	
	Invested	Return	Direct	Cross Terms
Invest1	33.0%	5.280%	0.206%	-0.072%
Invest2	33.0%	3.960%	0.062%	0.006%
Invest3	34.0%	2.720%	0.005%	-0.014%
	-----	-----		-----
Total	100.0%	11.960%		0.193%
Min probable rtn		3.173%		

If using an add-in optimizer then constrain the investment fractions to be > 0.0 and to add up to 1.0. If you want to use a minimum other than 0.0, e.g. constrain a money market fund at 10% to always preserve some liquidity, that's fine too. Also consider putting upper constraints on each fraction to prevent your portfolio from concentrating too much in a single investment. You may also wish to put a minimum acceptable value on the minimum probable return (e.g. 0% - no probable loss, or 4% - meet the average rate of inflation). Then have the optimizer maximize the Total Return. Run several cases with different allowed minimum probable returns to check the sensitivity of the solution.

- - - - -

The full equation for the variance calculation looks like this:

$$\begin{aligned}
 \text{var} &= i_1 * \text{var}_1 && \dots \\
 &+ i_2 * \text{var}_2 && \dots \text{ repeat for all investments} \\
 &+ i_3 * \text{var}_3 && \dots \\
 &+ 2*i_1*i_2*(\text{covar}_{1x2}) && \dots \\
 &+ 2*i_1*i_3*(\text{covar}_{1x3}) && \dots \text{ repeat for all possible pairs} \\
 &+ 2*i_2*i_3*(\text{covar}_{2x3}) && \dots
 \end{aligned}$$

Clearly this could be expressed via a matrix calculation if you prefer.

Other equations:

$$\text{Covariance.IxJ} = \text{Avg}(\text{Return.I} * \text{Return.J}) - \text{over all periods} \\ - \text{Avg}(\text{Return.I}) * \text{Avg}(\text{Return.J})$$

where the periods included are only those where data exists for both investments I and J.

$$\text{Minimum probable return} = \text{Return} - 2 * \text{Sqrt}(\text{Variance})$$

For 3 investments it is possible to do a manual optimization using 1-2-3 Data Tables. Following are 2 data tables that list the expected return and expected minimum return (a measure of risk) for various combinations of Invest1 and Invest2. In all cases Invest3 is calculated to force  $I1+I2+I3 = 1.0$ .

	Fraction		Variance calculation		
	Invested	Return	Direct	Cross Terms	
Invest1	0.30	0.0480	0.001872	-0.0005976	
Invest2	0.30	0.0360	0.000564	0.0000672	
Invest3	0.40	0.0320	0.000064	-0.0001536	
	-----	-----		-----	
Total	1.00	0.1160		0.001816	
Min probable rtn		0.0308			

Tables of results for various investment fractions

Table of Expected Return (I1 down, I2 across)

0.116	0.00%	10.00%	20.00%	30.00%	40.00%
0.00%	8.00%	8.40%	8.80%	9.20%	9.60%
10.00%	8.80%	9.20%	9.60%	10.00%	10.40%
20.00%	9.60%	10.00%	10.40%	10.80%	11.20%
30.00%	10.40%	10.80%	11.20%	11.60%	12.00%
40.00%	11.20%	11.60%	12.00%	12.40%	12.80%
50.00%	12.00%	12.40%	12.80%	13.20%	13.60%

Table of Expected Minimum Return (I1 down, I2 across)

0.030770897	0.00%	10.00%	20.00%	30.00%	40.00%
-------------	-------	--------	--------	--------	--------

0.00%	5.47%	4.49%	3.93%	3.57%	3.33%
10.00%	3.69%	3.49%	3.38%	3.34%	3.36%
20.00%	2.76%	2.86%	3.00%	3.19%	3.40%
30.00%	2.12%	2.41%	2.73%	3.08%	3.45%
40.00%	1.64%	2.07%	2.52%	3.00%	3.50%
50.00%	1.27%	1.80%	2.37%	2.95%	3.56%

Estimates of optimal portfolios for a given return taken from the above Tables

Return	Min Rtn	I1	I2	I3
8.00%	5.47%	0.00%	0.00%	100.00%
9.00%	3.75%	0.00%	25.00%	75.00%
10.00%	3.34%	10.00%	30.00%	60.00%
11.00%	3.30%	20.00%	35.00%	45.00%
12.00%	3.45%	30.00%	40.00%	30.00%
13.00%	3.76%	40.00%	45.00%	15.00%
14.00%	4.20%	50.00%	50.00%	0.00%

The above results assume that neither I1 or I2 is allowed to exceed 50% of the total portfolio. Examining the results shows that 100% I3 carries the lowest risk, but that significantly higher returns are predicted for only a small increase in risk with a 50/50 combination of I1/I2. That sort of result is exactly the kind of trade-off in risks that the theory is intended to point out. In this case it is because I1 and I2 have significant negative covariance, thus their risks tend to cancel out in the calculations and they both have a higher expected return than I3. For the data used here this outcome is nearly intuitive. For more investment choices and less unambiguous relationships the matter clouds considerably and the math can then help make an informed decision.

The author has used this basic approach and an Add-In optimizer on sets of up to 12 different investments.

- - - - -

-

-

-

-

-



-

-

-

-

-



- - -

50.00%  
10.00%  
10.80%  
11.60%  
12.40%  
13.20%  
14.00%

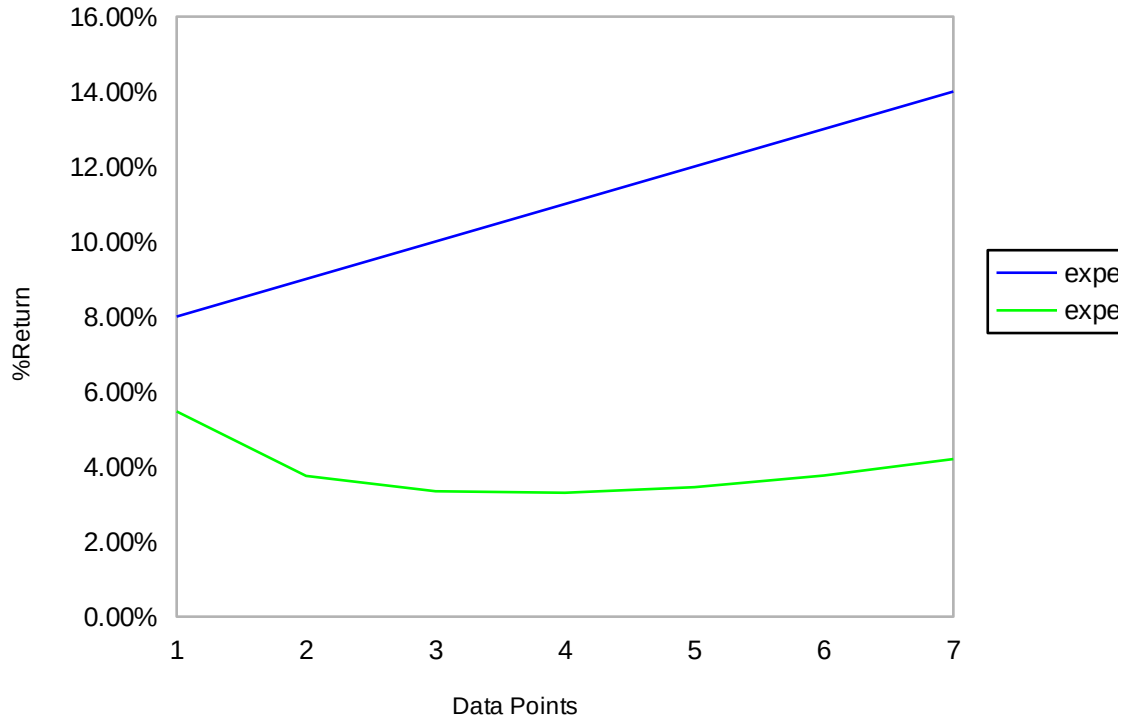
50.00%

3.19%  
3.43%  
3.65%  
3.85%  
4.03%  
4.20%

-

# Markowitz Portfolio Optimization

## Minimum Risk at a Given Return



cted_return
cted_min_return