Numeric Precision Requirements for Representation of a Location in a Global Coordinate System

The standard include file math.h when compiling in "non strict ascii" mode contains the following useful definitions:

M_E	2.7182818284590452354
M_LOG2E	1.4426950408889634074
M_LOG10E	0.43429448190325182765
M_LN2	0.69314718055994530942
M_LN10	2.30258509299404568402
M_PI	3.14159265358979323846
M_PI_2	1.57079632679489661923
M_PI_4	0.78539816339744830962
M_1_PI	0.31830988618379067154
M_2_PI	0.63661977236758134308
M_2_SQRTPI	1.12837916709551257390
M_SQRT2	1.41421356237309504880
M_SQRT1_2	0.70710678118654752440

The circumference of the Earth is approximately :

$$c_e = 24000 \text{ miles} * 1.609344\text{e}+03 \text{ meters/mile} = 38624256.000 \text{ meters}$$

From this we can see that any distance on the Earth can be expressed as a positive distance from any Origin on that surface with:

9 digits	to within 1 meter
12 digits	to within 1 millimeter

From the above constants, we can see that on a typical modern Unix workstation we can expect up to 21 significant digits in a C language double float, which ismore than sufficient for our requirements.

Universal Coordinate Systems

There are several different ways of expressing an accurate location in a 3-space, of which three are of particular interest here: Cartesian, Spherical and World Geographic.

Cartesian Coordinates

Cartesian coordinates are expressed as a triple of distances, $<x,y,z>$ where:

x = distance along x axis

y = distance along y axis

z = distance along z axis

Spherical Coordinates

Spherical coordinates are expressed as a triple with two angles in radians and one distance measurement, $<r,\phi,\theta>$ where:

ϕ = angle from z axis towards xy plane

θ = angle from x axis on xy plane

r = distance from center of sphere

and

$$0 <= \phi <= \pi$$

$$0 <= \theta < 2\pi$$

$$r >= 0$$

World Geographic Coordinates

World Geographical coordinates are expressed as a triple of latitude, longitude and altitude, where altitude is a distance and is often not included and angles are expressed in degrees, $<\phi_g, \theta_g, r_g>$ where:

ϕ_g = angle from the Equator towards a Pole, ie *Latitude*

ϕ_{gN} = angle from the Equator towards the North Pole

ϕ_{gS} = angle from the Equator towards the South Pole

λ_g = angle westwards from Greenwich Meridian along the equator, ie *Longitude*

ρ_g = distance from sea level of the Earth, ie *Altitude*

and

$$0 <= \phi_g <= 90^o$$

$$0 <= \lambda_g < 360^o$$

$$\rho_g => 0$$

Local Coordinate Systems

Due to historical reasons and matters of convenience, surface maps are often given 2d grid reference numbers that are related to a known local benchmark in a Cartesian coordinate system that is locally semi-congruent with a World Geographic Coordinate system. The congruency falls off with distance from the specified benchmark.

Grid systems in Ireland and the UK are usually given as a 12 figure Grid Reference. This is actually two numbers that give the cartesian coordinates in meters. Distances North and East of the origin are given as positive numbers as is usual for points on a cartesian plane.

The two local systems of most immediate concern are the *Irish Grid* which covers all of Ireland and the *National Grid* covering England, Scotland and Wales. These systems are not directly compatible, although the True Irish Grid Origin is directly related to the National Grid Origin. For convenience, Irish maps are given from a False Irish Grid Origin that is translated by <-200000,-200000> relative to the True Irish Grid Reference. This makes all Irish Grid Reference numbers positive.

Transformations between Coordinate Systems

The transformation between spherical coordinates and cartesian coordinates is:

$$x = r \sin\phi \cos\theta$$
$$y = r \sin\phi \sin\theta$$
$$z = r \cos\phi$$

World Geographic coordinates are simply related to spherical coordinates:

$$k_0 = 180/\pi \text{ degrees/radian}$$
$$r_g = r - r_e$$
$$\phi_{gN} = -k_0(\phi - \pi/2) \quad \text{for } 0 <= \phi < \pi/2$$
$$\phi_{gS} = k_0(\phi - \pi/2) \quad \text{for } \pi/2 <= \phi < \pi$$
$$\theta_g = k_0\theta$$