Introduction to Vector Theory:

One of the most important concerns in science and mathematics is the understanding of the special language shared by both fields. This language deals with the description of physical quantities and phenomenon. Essential to this description are the concepts of vector algebra.

Vectors and Scalars:

Some physical quantities can be described completely by a single number with its associated units. Such quantities include temperature, volume, time, mass, and density. Many other quantities, however, have a directional quality that cannot be described by a single number. These include velocity, force, and momentum.

A physical quantity that can be described completely by a single number is called a scalar quantity. A quantity that requires a direction as well as magnitude is called a vector. Calculations with scalar quantities use the operations of ordinary arithmetic, but calculations with vectors use operations that are somewhat different.

Notation for Vectors:

A vector is denoted by a bold faced letter such as **A**. In handwriting, vectors symbols are usually underlined or written with an arrow above. The magnitude is denoted with vertical bars or simply with the same letter in a light faced type.

$$(\text{Magnitude of } \mathbf{A}) = |\mathbf{A}| = A$$

Graphical Representation of a Vector:

A vector can be represented graphically using an arrow or directed line segment. The magnitude of the vector is determined by length of the arrow in appropriate units. The tail end of the arrow is called the initial point and the head is called the terminal point.

Mathematical Representation of a Vector:

A vector **A** can be represented with initial point at the origin of a rectangular coordinate system. If **i**, **j**, and **k** are unit vectors in the directions of the positive x, y, and z axes, then

$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$

where $A_x\mathbf{i}$, $A_y\mathbf{j}$, and $A_z\mathbf{k}$ are called component vectors of **A** in the **i**, **j**, and **k** directions and A_x, A_y, and A_z are called the components of **A**.

Fundamental Definitions of Vector Algebra:

1. Equality of vectors. Two vectors are equal if they have the same magnitude and direction. This is equivalent to two vectors having equal component values.

2. Multiplication of a vector by a scalar. If m is any real number (scalar), then m**A** is a vector whose magnitude is |m| times the magnitude of **A** and whose direction is the same as or opposite to **A** according to the sign of m. If m = 0, then m**A** = 0 is called the zero or null vector.

3. Sum of two vectors. The sum or resultant of **A** and **B** is a vector **C** = **A** + **B** formed graphically by placing the initial point of **B** on the terminal point of **A** and joining the initial point of **A** to the the terminal point of **B**. Note that this operation is not equivalent to simply adding the magnitudes of the two vectors; instead, the addition of vectors is accomplished most readily by adding the rectangular components of the vectors.

4. Unit vectors. A unit vector is a vector with a magnitude of one unit. If **A** is a vector, then a unit vector in the direction of **A** is **a** = **A**/|**A**|.

5. Dot product of two vectors. The dot product of two vectors is denoted by C = **A** · **B** and is defined as follows:

A · **B** = A B cos q = **|A|** **|B|** cos q

where q is the angle between the two vectors. The dot product is a scalar quantity, not a vector, and it may be either positive or negative. The dot product of two perpendicular vectors is always zero.

6. Cross product of two vectors. The cross product of two vectors is denoted by **C** = **A** x **B** and is defined as follows:

A x **B** = A B sin q **u** = **|A| |B|** sin q **u**

where q is the angle between the two vectors and **u** is a unit vector perpendicular to the plane of **A** and **B** such that **A**, **B**, and **u** form a right-handed system.

Laws and Formulas of Vector Algebra:

If **A**, **B**, **C** are vectors and m, n, are scalars, then

$\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$	Commutative law for vector addition
$\mathbf{A} + (\mathbf{B} + \mathbf{C}) = (\mathbf{A} + \mathbf{B}) + \mathbf{C}$	Associative law for vector addition
$m(n\mathbf{A}) = (mn)\mathbf{A} = n(m\mathbf{A})$	Associative law for scalar multiplication
$(m + n)\mathbf{A} = m\mathbf{A} + n\mathbf{A}$ $m(\mathbf{A} + \mathbf{B}) = m\mathbf{A} + m\mathbf{B}$	Distributive law for scalar multiplication
$\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}$	Commutative law for the dot product

$\mathbf{A} \cdot (\mathbf{B} + \mathbf{C}) = \mathbf{A} \cdot \mathbf{B} + \mathbf{A} \cdot \mathbf{C}$ Distributive law for the dot product

$\mathbf{A} \times \mathbf{B} = - \mathbf{B} \times \mathbf{A}$ Commutative property of the cross product

$$\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z$$

$$\mathbf{A} \times \mathbf{B} = (A_y B_z - A_z B_y)\mathbf{i} + (A_z B_x - A_x B_z)\mathbf{j} + (A_x B_y - A_y B_x)\mathbf{k}$$

$$\mathbf{A} \cdot (\mathbf{B}\ \acute{}\ \mathbf{C}) = A_x B_y C_z + A_y B_z C_x + A_z B_x C_y - A_z B_y C_x - A_y B_x C_z - A_x$$

$B_z C_y$

$|\mathbf{A} \cdot \mathbf{B}|$ = area of parallelogram having sides **A** and **B**

$|\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})|$ = volume of parallelepiped with sides **A**, **B**, and **C**

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}\,(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}\,(\mathbf{A} \cdot \mathbf{B})$$

$$(\mathbf{A} \times \mathbf{B}) \times \mathbf{C} = \mathbf{B}\,(\mathbf{A} \cdot \mathbf{C}) - \mathbf{A}\,(\mathbf{B} \cdot \mathbf{C})$$

$$(\mathbf{A} \times \mathbf{B}) \cdot (\mathbf{C} \times \mathbf{D}) = (\mathbf{A} \cdot \mathbf{C})(\mathbf{B} \cdot \mathbf{D}) - (\mathbf{A} \cdot \mathbf{D})(\mathbf{B} \cdot \mathbf{C})$$

$$(\mathbf{A} \times \mathbf{B}) \times (\mathbf{C} \times \mathbf{D}) = \mathbf{C}\,\{\mathbf{A} \cdot (\mathbf{B} \times \mathbf{D})\} - \mathbf{D}\,\{\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})\}$$
$$= \mathbf{B}\,\{\mathbf{A} \cdot (\mathbf{C} \times \mathbf{D})\} - \mathbf{A}\,\{\mathbf{B} \cdot (\mathbf{C} \times \mathbf{D})\}$$