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1 Introduction

This paper contains examples of various features from -LATEX.

2 Enumeration of Hamiltonian paths in a graph

Let be the adjacency matrix of graph G. The corresponding Kirchhoff matrix is obtained from **A** by replacing in ?**A** each diagonal entry by the degree of its corresponding vertex; i.e., the ith diagonal entry is identified with the degree of the ith vertex. It is well known that

$$\det\mathbf{K}(i|i)=\mathit{thenumberofspanningtreesofG},\mathrm{i=1,?,n} \tag{1}$$

where $\mathbf{K}(i|i)$ is the ith principal submatrix of **K**.

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\det\mathbfK(i|i)=\text the number of spanning trees of
$G$,
```

Let be the set of graphs obtained from G by attaching edge to each spanning tree of G. Denote by . It is obvious that the collection of Hamiltonian cycles is a subset of . Note that the cardinality of is . Let .

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$\wh X=\\hat x_1,\dots,\hat x_n\$
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Define multiplication for the elements of by

(2)

Let and . Then the number of Hamiltonian cycles is given by the relation [liuchow\s\do5(f)ormalsum]

(3)

The task here is to express (?) in a form free of any , i=1,?,n. The result also leads to the resolution of enumeration of Hamiltonian paths in a graph.

It is well known that the enumeration of Hamiltonian cycles and paths in a complete graph and in a complete bipartite graph can only be found from *first combinatorial principles* [hapa\s\do5(g)raphenum]. One wonders if there exists a formula which can be used very efficiently to produce and . Recently, using Lagrangian methods, Goulden and Jackson have shown that can be expressed in terms of the determinant and permanent of the adjacency matrix [gouja\s\do5(l)agrmeth]. However, the formula of Goulden and Jackson determines neither nor effectively. In this paper, using an algebraic method, we parametrize the adjacency matrix. The resulting formula also involves the determinant and permanent, but it can easily be applied to and . In addition, we eliminate the permanent from and show that can be represented by a determinantal function of multivariables, each variable with domain 0,1. Furthermore, we show that can be written by number of spanning trees of subgraphs. Finally, we apply the formulas to a complete multigraph .

The conditions , i,j=1,?,n, are not required in this paper. All formulas can be extended to a digraph simply by multiplying by 2.

3 Main Theorem

[Sorry. Ignored `\beginnotation ... \endnotation`]