

## ◇ Choice between processes ◇

We have used  $|$  and menu choice to describe processes which have alternative behaviours. We have emphasised that  $|$  is *not* an operation on processes, but can only be used in conjunction with distinct prefixing events.

However, CSP does have operators which can be used to provide a choice between two (or more) existing processes. They are:

*external choice* - the environment can choose between the various processes

*internal choice* - the choice is made within the process, and cannot be observed by the environment.

By *the environment*, we mean whatever processes are in parallel with the process containing the choice.

The distinction between choice made by a process and choice made by its environment is important, because problems could arise if two processes have both been given control over a particular choice.

## ◇ External Choice ◇

The process  $P \sqcap Q$  (pronounced “ $P$  external choice  $Q$ ”) is initially prepared to do any event which either  $P$  or  $Q$  could do. After the first event, the behaviour is either that of  $P$  or that of  $Q$ , depending on which process did the event. The choice is called “external” because the environment (another process in parallel) can choose the first event.

*Example:* The journey from A (the bus station) to B is covered by two bus routes: the 37 and the 111. If both buses are present at the bus station, then the service offered to the passenger is described by the process

$$SERVICE = BUS37 \sqcap BUS111.$$

The passenger can choose which bus to use.

Here are possible definitions:

$$\begin{aligned} BUS37 = & \\ & board.37.A \rightarrow (pay.90 \rightarrow alight.37.B \rightarrow Stop \\ & \quad | alight.37.A \rightarrow Stop) \end{aligned}$$

$$\begin{aligned} BUS111 = & \\ & board.111.A \rightarrow (pay.70 \rightarrow alight.111.B \rightarrow Stop \\ & \quad | alight.111.A \rightarrow Stop) \end{aligned}$$

Note that in this case, we do not think of events such as *alight.111.B* as related to input or output.

If the passenger is defined by

$$PASS = board.37.A \rightarrow pay.90 \rightarrow alight.37.B \rightarrow Stop$$

then we can consider what happens when the passenger and the bus service interact, i.e. when we construct

$$SERVICE \alpha SERVICE \parallel_{\alpha} PASS \text{ } PASS.$$

*SERVICE* can behave either as *BUS37* or as *BUS111*, and the choice is made by the environment. The fact that *PASS* can only do *board.37* as its first event, means that *BUS37* is chosen.

The system behaves exactly as if we had written

$$\begin{aligned} SERVICE \\ = & board.37.A \rightarrow (pay.90 \rightarrow alight.37.B \rightarrow Stop \\ & \quad | alight.37.A \rightarrow Stop) \\ | & board.111.A \rightarrow (pay.70 \rightarrow alight.111.B \rightarrow Stop \\ & \quad | alight.111.A \rightarrow Stop) \end{aligned}$$

In general,  $(a \rightarrow P) \square (b \rightarrow Q)$  is equivalent to  $a \rightarrow P \mid b \rightarrow Q$ , and it is possible to use  $\square$  instead of  $\mid$  (this is what FDR does).

However, we can also write  $(a \rightarrow P) \square (a \rightarrow Q)$  (remember that  $a \rightarrow P \mid a \rightarrow Q$  is illegal) — we will see what this means soon.

## ◇ Defining External Choice ◇

Here are the transition rules for external choice.

$$\begin{array}{c} \frac{P \xrightarrow{a} P'}{P \square Q \xrightarrow{a} P'} \qquad \frac{Q \xrightarrow{a} Q'}{P \square Q \xrightarrow{a} Q'} \\[10pt] \frac{P \xrightarrow{\tau} P'}{P \square Q \xrightarrow{\tau} P' \square Q} \qquad \frac{Q \xrightarrow{\tau} Q'}{P \square Q \xrightarrow{\tau} P \square Q'} \end{array}$$

The first two capture the intention that the choice is resolved by the first event. The second two allow either process to change state internally without resolving the choice.

*Example:* Going back to

$$SERVICE = BUS37 \square BUS111$$

we have the transitions

$$\begin{aligned} SERVICE & \xrightarrow{board.37.A} \\ & pay.90 \rightarrow \dots \mid alight.37.A \rightarrow Stop \end{aligned}$$

$$\begin{aligned} SERVICE & \xrightarrow{board.111.A} \\ & pay.70 \rightarrow \dots \mid alight.111.A \rightarrow Stop. \end{aligned}$$

## ◇ Internal Choice ◇

The process  $P \sqcap Q$  describes a choice between  $P$  and  $Q$ , but the environment has no control over the choice. Internal choice is often also known as *non-deterministic choice*. The choice is resolved internally by the process.

Suppose the bus company agrees to provide a bus from A to B, but does not say whether it will be the 37 or the 111. The situation at the bus station is now described by the process

$$SERVICE = BUS37 \sqcap BUS111.$$

We should interpret this as a specification of a bus service. The company could implement the service by always providing bus 37, or by deciding each morning which bus to provide, etc. The passenger has no control over the decision, and cannot tell which bus will be available until she arrives at the bus station.

If a system is specified by the description  $P \sqcap Q$ , then all of the following are acceptable implementations.

- ◇ provide both  $P$  and  $Q$ , and use some internal means to choose between them
- ◇ just provide  $P$
- ◇ just provide  $Q$

## ◇ Internal Choice ◇

To define internal choice by means of transition rules, we use the *internal event*  $\tau$ . A transition  $P \xrightarrow{\tau} Q$  represents a change of state which is not accompanied by any observable event; it is a change of state whose occurrence cannot be observed directly by the environment. We use  $\tau$  transitions to model the resolution of an internal choice.

Here are the transition rules:

$$\frac{}{P \sqcap Q \xrightarrow{\tau} P} \qquad \frac{}{P \sqcap Q \xrightarrow{\tau} Q}$$

Note that these rules capture one approach to implementing  $P \sqcap Q$ , namely to implement both  $P$  and  $Q$  and then choose between them at random. In order to give transition rules we are forced to choose an implementation, and this is the most general.

### ◇ Example ◇

Consider

$$SERVICE = BUS37 \sqcap BUS111$$

again, and put it in parallel with *PASS*. According to the transition rules, the first event which *SERVICE* does will be a  $\tau$  event, resulting in either *BUS37* or *BUS111*. All the events of *PASS* require synchronisation, so nothing can happen until  $\tau$  has been done.

There are two ways for *SERVICE* to do  $\tau$ . The first results in

$$BUS37 \parallel_{\alpha} SERVICE \parallel_{\alpha} PASS \parallel_{\alpha} PASS$$

and then *PASS* can interact with *BUS37*.

The other possibility results in

$$BUS111 \parallel_{\alpha} SERVICE \parallel_{\alpha} PASS \parallel_{\alpha} PASS$$

and now the whole system stops because *BUS111* and *PASS* cannot synchronise on any events. This is another example of *deadlock*.

### ◇ Another example ◇

Keep the definition

$$SERVICE = BUS37 \sqcap BUS111$$

and suppose that there is also a train service from A to B, described by the process *TRAIN*. Now the options available to the passenger are described by the process

$$TRAIN \sqcap SERVICE$$

which expands to

$$TRAIN \sqcap (BUS37 \sqcap BUS111).$$

We have the transition

$$BUS37 \sqcap BUS111 \xrightarrow{\tau} BUS37$$

and so the transition rules for external choice give

$$TRAIN \sqcap (BUS37 \sqcap BUS111)$$

$$\downarrow \tau$$

$$TRAIN \sqcap BUS37$$

We can interpret this transition as the fact that one bus service may disappear while the passenger is still thinking about whether to take the bus or the train.

If the definition of *TRAIN* is

$$TRAIN = board.train.A \rightarrow alight.train.B \rightarrow Stop$$

then there is also the transition

$$TRAIN \sqcap (BUS37 \sqcap BUS111)$$

|  
*board.train.A*

$$alight.train.B \rightarrow Stop$$

which we can interpret as the passenger choosing the train without ever discovering which bus is available.

## ◇ Nondeterminism ◇

The first form of choice,  $|$ , is a special case of external choice. The process

$$a \rightarrow P \mid b \rightarrow Q$$

is equivalent to

$$a \rightarrow P \sqcap b \rightarrow Q.$$

However, general external choice has some extra power. Because it is possible to construct an external choice between any two processes, we can write, for example

$$a \rightarrow P \sqcap a \rightarrow Q$$

(recall that  $a \rightarrow P \mid a \rightarrow Q$  is forbidden).

We consider  $\rightarrow$  to have higher precedence than  $\sqcap$ , so that this process is the same as

$$(a \rightarrow P) \sqcap (a \rightarrow Q).$$

What does this mean? The process

$$a \rightarrow P \sqcap a \rightarrow Q$$

can either do  $a$  and then behave like  $P$ , or do  $a$  and behave like  $Q$ . The environment cannot influence which of these possibilities will occur: all it can do is choose to do  $a$  in order to interact.

More generally, the external choice

$$a \rightarrow P \sqcap a \rightarrow Q \sqcap b \rightarrow R$$

allows the environment to choose between  $a$  and  $b$ , but if  $a$  is chosen then the subsequent behaviour could be that of either  $P$  or  $Q$ .

Using external choice with several occurrences of the same prefixing event leads to nondeterminism, in the sense that the event which is observed does not determine the subsequent behaviour.

We will eventually see that

$$a \rightarrow P \sqcap a \rightarrow Q = a \rightarrow P \sqcap a \rightarrow Q$$

which emphasises the fact that the environment cannot choose between  $P$  and  $Q$ .